Spatially Close Signals Separation via Array Aperture Expansions and Spatial Spectrum Averaging

Heung Yong Kang, Young Su Kim, and Chang Joo Kim

ABSTRACT—A resolution enhancement method for estimating the direction-of-arrival (DOA) of signals is presented. The proposed method is by virtually expanding a real array into virtual arrays and then averaging the spatial spectrum of the virtual arrays, each of which has a different aperture size. Superior DOA resolutions are shown in comparison with the standard algorithm, MUltiple SIgnal Classification (MUSIC), for incoherent signals incident on a uniform circular array.

Keywords—DOA estimation, MUSIC, array antenna.

I. Introduction

The direction-of-arrival (DOA) resolution of signals incident on an array depends on the aperture size of the array, the number of sensors, the number of snapshots, and the signal-to-noise ratio (SNR) [1]. One of the methods to improve resolution is to increase the aperture size of the array. However, it is impractical to increase the physical size of the array in a real environment. Improved DOA resolution using the virtual expansion of an array was proposed in [2]. But increasing the aperture size of the array without increasing the number of sensors results in spurious peaks of spatial spectrum, which may cause a failure in estimating the correct DOA.

We propose a new method that expands the real array into virtual arrays with different aperture sizes, and then averages the spatial spectrums of the virtual arrays to reduce spurious peaks.

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II. Virtual Array Expansion

When *N* signals are incident on a general array of *M* identical sensors, the covariance matrix of a general array, $\mathbf{R}_x$, is given by

$$\mathbf{R}_x = \mathbf{A}(\boldsymbol{\kappa})\mathbf{R}_s\mathbf{A}(\boldsymbol{\kappa})^H + \sigma^2\mathbf{R}_\eta , \qquad (1)$$

where $\mathbf{R}_s$ is the covariance matrix of incident signals, $\sigma^2\mathbf{R}_\eta$ is the covariance matrix of additive noise, $\mathbf{A}(\boldsymbol{\kappa})$ is the $M \times N$ steering matrix of *N* incident signals, and $\boldsymbol{\kappa}$ is the unit direction vectors of *N* incident signals.

Consider a steering matrix, $\mathbf{B}(\boldsymbol{\kappa}, h)$, for a virtual array which is expanded *h* times as large as the original array. If $\mathbf{A}(\boldsymbol{\kappa})$ and $\mathbf{B}(\boldsymbol{\kappa}, h)$ have full rank of *N*, then there exists the M×M unitary transformation matrix, $\mathbf{T}(\boldsymbol{\kappa}, h)$, such that

$$\mathbf{B}(\boldsymbol{\kappa}, h) = \mathbf{T}(\boldsymbol{\kappa}, h)\mathbf{A}(\boldsymbol{\kappa}). \qquad (2)$$

The proof of (2) is given in [3]. From (2), the covariance matrix of the virtually expanded array, $\mathbf{R}_y$, is given by

$$\mathbf{R}_y = \mathbf{T}(\boldsymbol{\kappa}, h)\mathbf{R}_x\mathbf{T}(\boldsymbol{\kappa}, h)^H . \qquad (3)$$

By estimating $\mathbf{T}(\boldsymbol{\kappa}, h)$ and computing $\mathbf{R}_y$, the existing DOA algorithm such as MUltiple SIgnal Classification (MUSIC) can be applicable for the virtually expanded array [2].

III. Averaged Spatial Spectrum of Virtual Arrays

The spatial spectrums of virtually expanded arrays show spurious peaks because the sensor spacing becomes larger than the original array without increasing the number of sensors. It is very difficult to discriminate the real peaks from these spurious peaks when the magnitude of spurious peaks is high. The magnitudes and occurrences of spurious peaks are proportional to the aperture size of the array. Therefore we can only increase the aperture size of a virtual array to a certain extent when we use only one virtual array. But if we use several virtual arrays, each with a different expanding factor *h*, then we can solve this problem.

The positions and magnitudes of spurious peaks of a virtual array depend on the expanding factor *h* because the locations of the sensors are changed by the expanding factor. If we average the spatial spectrums of the virtual arrays, which have different expanding factors from each other, then the averaged spatial spectrum shows reduced spurious peaks. The averaged spatial spectrum of virtually expanded arrays, $P_{av}(\boldsymbol{\kappa})$, can be represented as follows:

$$P_{av}(\boldsymbol{\kappa}) = \frac{1}{Q}\sum_{i=1}^{Q} P_i(\boldsymbol{\kappa}), \tag{4}$$

where Q is the number of virtually expanded arrays, and $P_i(\boldsymbol{\kappa})$ is the spatial spectrum of *i*-th virtual array which has the expanding factor h_i.

IV. Simulation Results

To show the effectiveness of the proposed methods in resolving closely spaced plane waves, the case of two narrowband incoherent sources incident on a uniform circular array with 8 sensors is considered. The radius of the original array is 0.5 λ. Nine virtual arrays are used to enhance the resolution capability and the expanding factors of the virtual arrays are *h*=[9,10,11,12,13,14,15,16,17]. In case of narrow band sources, a Medium Description Length or Akaike's Information Criterion method can be used in determining the number of sources. But we assumed that the number of sources is known and used MUSIC as a spectrum estimation algorithm. The antenna noise vector is taken as a complex, valued additive, white Gaussian process whose components have identical variances and are statistically independent of the source signals.

Two incident plane waves are taken to have bearing angles $\theta_1 = -3.0°$ and $\theta_2 = 3.0°$. The antenna noise variance is selected so that the SNR of 5 dB is obtained. In employing the proposed methods, a set of initial angles is chosen to be {–3.5°, –3.0°, –2.5°, –2.0°, 2.0°, 2.5°, 3.0°, 3.5°}, which is the preliminary estimate angle [3]. This experiment was repeated ten times and the resulted ten spectral estimates are shown in superimposed fashion in Figs. 1(a), (b) and (c). From these estimates, it is evident that the MUSIC spectrum of the original array fails to resolve the two signals in any of the ten trial runs, while the MUSIC spectrum of a single virtual array with *h*=17 partially

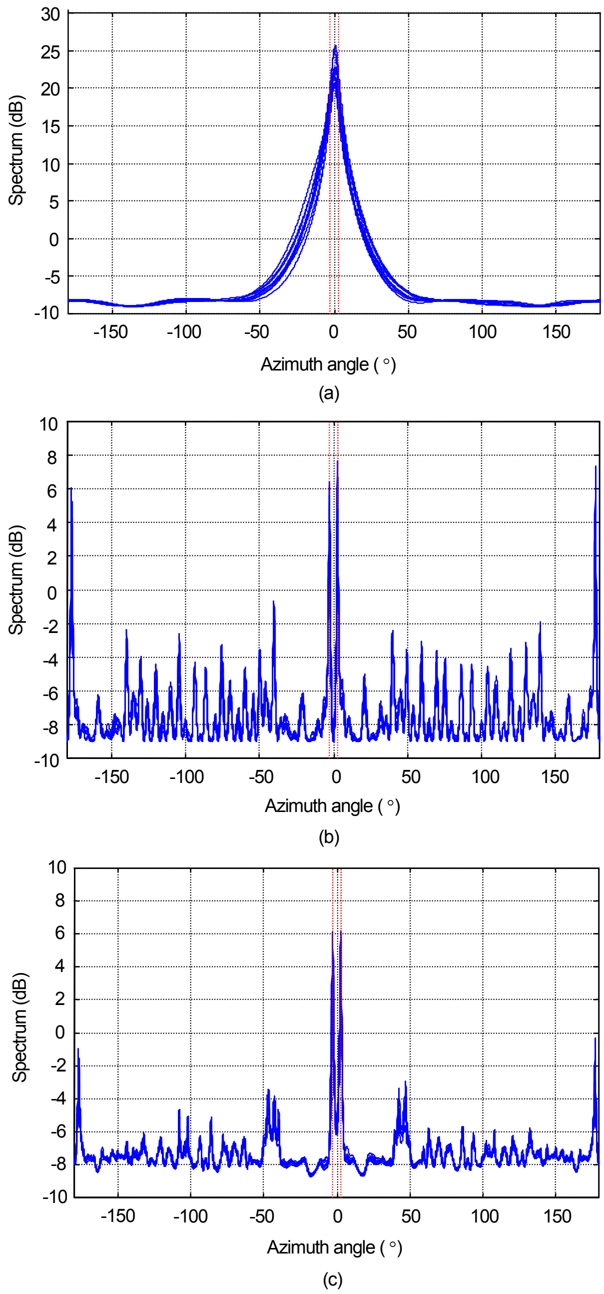

Fig. 1. Ten statistically independent superimposed bearing estimates for two incoherent plane waves at azimuth angles of –3.0° and 3.0°: (a) spatial spectrum of original array (*h*=1, SNR=5 dB) (b) spatial spectrum of virtually expanded array (*h*=17, SNR=5 dB) (c) averaged spatial spectrum of virtually expanded arrays (*h*=[9,10,11,12,13,14,15,16,17], SNR=5 dB).

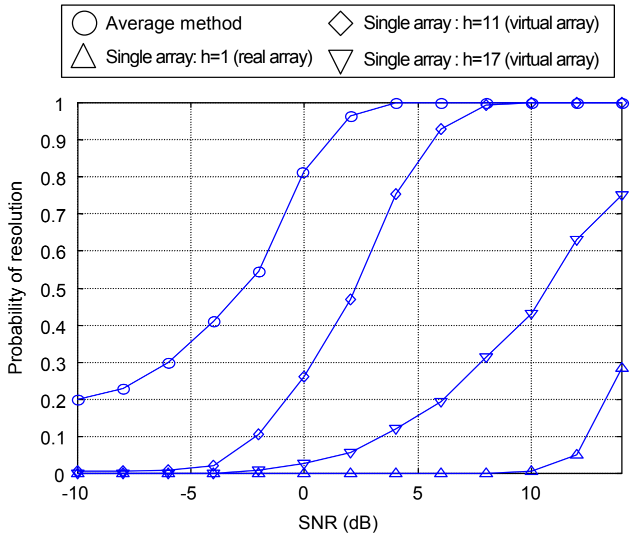

Fig. 2. Probability of resolution versus SNR (azimuth=[–3.0˚, 3.0˚])

resolves the two signals. The averaged MUSIC spectrum of the nine virtual arrays, however, achieves a consistent resolution and has small spurious peaks compared with the case of the single virtual array.

To investigate the comparative resolution capability of the proposed method, we carried out a computer simulation with the SNR altered. Two hundred independent trials were made. In this case, the azimuth angles of the sources were {–3.0°, 3.0°} and the preliminary estimates of the azimuth angles were taken to be {–3.5°, –3.0°, –2.5°, –2.0°, 2.0°, 2.5°, 3.0°, 3.5°}.

The probability of resolution is herein defined as the probability that both the first and second sources are estimated in the interval [–3.0°–2.0°, 3.0°+2.0°]. Figure 2 shows the probability of resolution when the SNR is changed. From the results of Fig. 2, we know that the averaged spectrum method with nine virtual arrays has a better resolution performance than the virtual expansion method with a single array.

V. Conclusions

A new algorithm has been presented for improving the resolution capability of sensor arrays on which multiple narrowband incoherent sources are impinging. The fundamental concept is based on the virtual expansion of the original array and the process of averaging the spatial spectrums of the virtual arrays which have different aperture sizes. From the simulation results, we found that the proposed algorithms provide superior resolution performance relative to that of standard MUltiple SIgnal Classification.

References

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