General Coupling Matrix Synthesis Method for Microwave Resonator Filters of Arbitrary Topology

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ABSTRACT—This letter presents a new approach to synthesize the resonator filters of an arbitrary topology. This method employs an optimization method based on the relation between the polynomial coefficients of the transfer function and those of the S_{21} from the coupling matrix. Therefore, this new method can also be applied to self-equalized filters that were not considered in the conventional optimization methods. Two microwave filters, a symmetric 4-pole filter with four transmission zeros (TZs) and an asymmetric 8-pole filter with seven TZs, are synthesized using the present method for validation. Excellent agreement between the response of the transfer function and that of the synthesized S_{21} from the coupling matrix is shown.

Keywords—Microwave filter, coupling matrix, synthesis.

I. Introduction

In wireless communication systems, stringent specifications of filters such as high frequency selectivity and group delay equalization are required to meet efficient spectrum utilization and to reduce the distortion in a digital data transmission. A modern high-performance filter demands a new topology of the coupling network containing the finite transmission zeros (TZs) because the number of TZs is directly related to frequency selectivity and group delay equalization.

In [1], a synthesis method of a 4-pole filter with two TZs was proposed by solving the nonlinear equations based on the relation between the coefficients of the transfer function and coupling matrix. This method is very useful for a 4-pole filter, but no other equations were provided for an arbitrary topology. The efficient synthesis methods using a coupling matrix rotation for more than a 4th-degree filter were proposed in [2]-[5]. However, an appropriate approach should be obtained individually for a new topology because there is no general rule for determining the sequence of matrix rotations. Unfortunately, it is difficult to derive the equations to solve for rotation angle analytically. In [6], a synthesis method was proposed using a gradient-based optimization technique with simple cost function. Since the amplitude of S_{21} and S_{11} was evaluated only at the critical frequencies such as $\pm j1$, zeros, and poles, this approach is effective to synthesize the resonator filters containing pure imaginary TZs for high-frequency selectivity. However, the goal function including information about the phase of S_{21} is not provided, and there is no consideration for group delay equalization. Therefore, a general coupling matrix synthesis approach is required with respect to the given transfer function containing both pure imaginary and complex zeros for high-frequency selectivity and group delay equalization.

In this letter, we propose a new approach to synthesize the resonator filters of an arbitrary topology. First, the simple formulas to compute the polynomial coefficients of the S_{21} from the coupling matrix are derived. Comparison between the polynomial coefficients of the target transfer function and those of the S_{21} from the coupling matrix gives us the exact goal function with an arbitrary frequency characteristic containing both pure imaginary TZs and complex TZs. To validate this method using this goal function, a symmetric 4-pole filter and an asymmetric 8-pole filter are synthesized. Excellent agreement between the response of the transfer function and that of the S_{21} from the coupling matrix is shown.

II. Synthesis Method

Generally, the transfer function of the filter is written as

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$$S_{21}(s) = P(s) \big/ \varepsilon E(s), \tag{1}$$

where ε is a ripple factor, s is a complex frequency variable, $E(s)$ is an N-th-degree Hurwitz polynomial, N is the degree of the filtering function, and $P(s)$ is the characteristic polynomial containing the TZs.

Figure 1 shows the coupling scheme of the general N-coupled filter network with source/load multi-resonator coupling. The transmission coefficient S_{21} is given as in [6] as

$$S_{21} = -2j[A^{-1}]_{N+2,1}. \tag{2}$$

Here, A is a $(N+2) \times (N+2)$ matrix containing complex frequency variable and frequency-independent coupling coefficients, $M_{p,q}$.

$$A = -j \cdot \begin{bmatrix} 1 & jM_{S,1} & jM_{S,2} & \cdots & jM_{S,N} & jM_{S,L} \\ jM_{S,1} & s+jM_{1,1} & jM_{1,2} & \cdots & jM_{1,N} & jM_{1,L} \\ jM_{S,2} & jM_{1,2} & s+jM_{2,2} & \cdots & jM_{2,N} & jM_{2,L} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ jM_{S,N} & jM_{1,N} & jM_{2,N} & \cdots & s+jM_{N,N} & jM_{N,L} \\ jM_{S,L} & jM_{1,L} & jM_{2,L} & \cdots & jM_{N,L} & 1 \end{bmatrix}$$

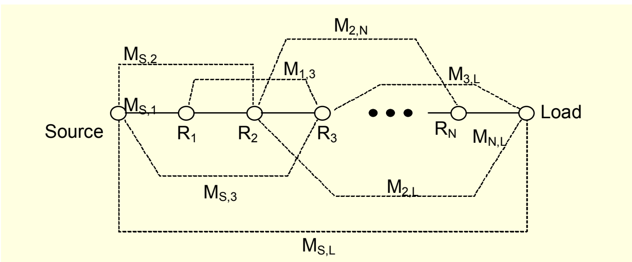

Fig. 1. Coupling and routing diagram of N-coupled filter network. R: resonator, M: mutual coupling, solid lines: direct coupling, dashed lines: cross coupling.

The transmission coefficient defined in (2) can be expressed in terms of the denominator and numerator polynomials as

$$S_{21} = \frac{N(s)}{D(s)} = \sum_{i=0}^{N} N_i \cdot s^i \Big/ \sum_{i=0}^{N} D_i \cdot s^i, \tag{3}$$

where D_i and N_i are the coefficients of denominator and numerator polynomials, respectively. By comparing (2) and (3), it is known that the determinant of A is identical to the denominator polynomial $D(s)$.

The coefficients of $D(s)$ can be obtained using the i-th derivative of the determinant of A as

$$D_i = \frac{1}{i!} \frac{d^i}{ds^i} |A| \Bigg|_{s=0}. \tag{4}$$

Here, the derivative of the determinant of A with respect to s is the sum of $N+2$ determinants obtained by replacing the elements of each row (column) by their derivatives with respect to s. The coefficients of a polynomial for the matrix whose element s exists in only the diagonal elements can be expressed as the sum of the principal minors [7]. Therefore, the coefficients of the denominator polynomial can be formulated simply as summarized in (5).

The numerator of the transmission coefficient, shown in (2), can be obtained from the determinant of the matrix of which the elements of the $(N+2)$th row and the first column are removed. First of all, s should be put in the diagonal elements to apply the similar method for the denominator. After moving s to the diagonal element by interchanging its row (column), we can obtain the coefficients of the numerator polynomial as (6) using the similar approach for the denominator polynomials.

$$D_N = \left(1 + M_{S,L}{}^2\right)$$

$$D_{N-1} = \sum_{i_3=1}^{N} \begin{vmatrix} 1 & jM_{S,L} & jM_{S,i_3} \\ jM_{S,L} & 1 & jM_{i_3,L} \\ jM_{S,i_3} & jM_{i_3,L} & jM_{i_3,i_3} \end{vmatrix}$$

$$\vdots$$

$$D_1 = \sum_{i_3=1}^{2} \sum_{i_4=i_3+1}^{3} \cdots \sum_{i_N=i_{N-1}+1}^{N-1} \sum_{i_{N+1}=i_N+1}^{N} \begin{vmatrix} 1 & jM_{S,L} & jM_{S,i_3} & \cdots & jM_{S,i_N} & jM_{S,i_{N+1}} \\ jM_{S,L} & 1 & jM_{i_3,L} & \cdots & jM_{i_N,L} & jM_{i_{N+1},L} \\ jM_{S,i_3} & jM_{i_3,L} & jM_{i_3,i_3} & \cdots & jM_{i_3,i_N} & jM_{i_3,i_{N+1}} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ jM_{S,i_N} & jM_{i_N,L} & jM_{i_3,i_N} & \cdots & jM_{i_N,i_N} & jM_{i_N,i_{N+1}} \\ jM_{S,i_{N+1}} & jM_{i_{N+1},L} & jM_{i_3,i_{N+1}} & \cdots & jM_{i_N,i_{N+1}} & jM_{i_{N+1},i_{N+1}} \end{vmatrix}$$

$$D_0 = \sum_{i_3=1}^{1} \sum_{i_4=i_3+1}^{2} \cdots \sum_{i_{N+1}=i_N+1}^{N-1} \sum_{i_{N+2}=i_{N+1}+1}^{N} \begin{vmatrix} 1 & jM_{S,L} & jM_{S,i_3} & \cdots & jM_{S,i_{N+1}} & jM_{S,i_{N+2}} \\ jM_{S,L} & 1 & jM_{i_3,L} & \cdots & jM_{i_{N+1},L} & jM_{i_{N+2},L} \\ jM_{S,i_3} & jM_{i_3,L} & jM_{i_3,i_3} & \cdots & jM_{i_3,i_{N+1}} & jM_{i_3,i_{N+2}} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ jM_{S,i_{N+1}} & jM_{i_{N+1},L} & jM_{i_3,i_{N+1}} & \cdots & jM_{i_{N+1},i_{N+1}} & jM_{i_{N+1},i_{N+2}} \\ jM_{S,i_{N+2}} & jM_{i_{N+2},L} & jM_{i_3,i_{N+2}} & \cdots & jM_{i_{N+1},i_{N+2}} & jM_{i_{N+2},i_{N+2}} \end{vmatrix} \tag{5}$$

$$N_N = -2 \cdot jM_{S,L}$$

$$N_{N-1} = -2 \cdot \sum_{i_2=1}^{N} \begin{vmatrix} jM_{S,L} & jM_{i_2,L} \\ jM_{S,i_2} & jM_{i_2,i_2} \end{vmatrix}$$

$$\vdots$$

$$N_0 = -2 \cdot \sum_{i_2=1}^{1} \sum_{i_3=i_2+1}^{2} \cdots \sum_{i_{N+1}=i_N+1}^{N} \begin{vmatrix} jM_{S,L} & jM_{i_2,L} & jM_{i_3,L} & \cdots & jM_{i_N,L} & jM_{i_{N+1},L} \\ jM_{S,i_2} & jM_{i_2,i_2} & jM_{i_2,i_3} & \cdots & jM_{i_2,i_N} & jM_{i_2,i_{N+1}} \\ jM_{S,i_3} & jM_{i_2,i_3} & jM_{i_3,i_3} & \cdots & jM_{i_3,i_N} & jM_{i_3,i_{N+1}} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ jM_{S,i_N} & jM_{i_2,i_N} & jM_{i_3,i_N} & \cdots & jM_{i_N,i_N} & jM_{i_N,i_{N+1}} \\ jM_{S,i_{N+1}} & jM_{i_2,i_{N+1}} & jM_{i_3,i_{N+1}} & \cdots & jM_{i_N,i_{N+1}} & jM_{i_{N+1},i_{N+1}} \end{vmatrix} \tag{6}$$

The coefficients of the denominator polynomials and the numerator polynomials given by (5) and (6) should be equal to the coefficients of the polynomials given by (1), respectively. Thus, the goal function for an arbitrary topology of the coupling network can be obtained by comparing the polynomial coefficients from the coupling matrix and those of the transfer function. The goal function can be expressed as

$$G=\sum_{i=0}^{N-1}\left|E_i-\frac{D_i}{D_N}\right|+\sum_{i=0}^{N}\left|\frac{P_i}{\varepsilon}-\frac{N_i}{D_N}\right|. \qquad (7)$$

III. Examples

To verify this present synthesis method, it is applied to a symmetric 4-pole filter and an asymmetric 8-pole filter with high frequency selectivity and group delay equalization.

The 4-pole filter has two TZs ($\pm j3.6$) for high frequency selectivity and two TZs (± 0.9805) for group delay equalization in the pass-band. The transfer function of this filter is given as

$$t(s)=\frac{1}{\varepsilon}\frac{s^4+P_2s^2+P_0}{s^4+E_3s^3+E_2s^2+E_1s+E_0}, \qquad (8)$$

where E_3=2.48762, E_2=4.11089, E_1=3.45246, E_0=1.15726
P_2=11.99860, $P_0=-12.45950$, $\varepsilon=-j\,10.75060$.

From (6), we know that the coupling value ($M_{S,L}$) between source and load should be non-zero in order to have four TZs in a 4-pole filter. The coupling scheme of the filter is shown in Fig. 2.

A useful optimization method (solving a set of non-linear equations in Mathcad®) is applied to the filter. Because the initial couplings in the optimization method depend on its convergence, the appropriate values have to be chosen. First of all, the initial couplings are chosen from the known coupling matrix of a similar configuration filter. Before operating the optimization, the values are adjusted by comparing between the sign of the coefficients of the transfer function and the coefficients obtained from (5) and (6). The coupling values obtained by optimization are shown in (9). Figure 3 shows the frequency response of the transfer function in (8) and the coupling matrix (M) given by (9).

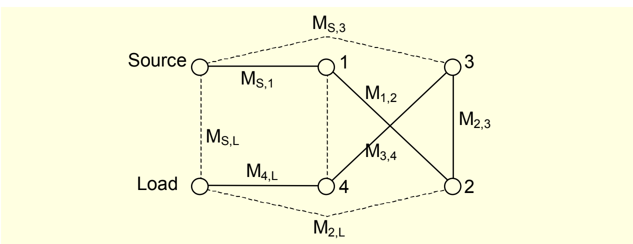

Fig. 2. Coupling scheme of a 4-pole filter with four TZs.

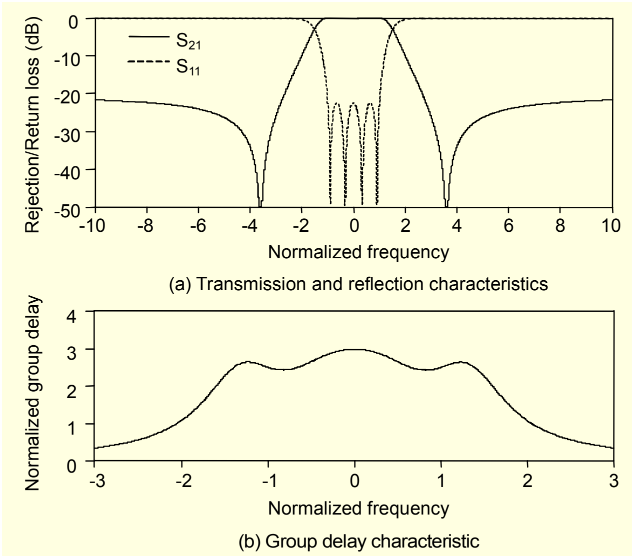

Fig. 3. 4-pole filter with four TZs. The results agree within the plotting accuracy and cannot be distinguished.

$$M=\begin{bmatrix} 0 & 1.1060 & 0 & -0.1528 & 0 & -0.0466 \\ 1.1060 & 0 & 1.002 & 0 & 0.6298 & 0 \\ 0 & 1.002 & 0 & 0.3397 & 0 & -0.1528 \\ -0.1528 & 0 & 0.3397 & 0 & 1.002 & 0 \\ 0 & 0.6298 & 0 & 1.002 & 0 & 1.1060 \\ -0.0466 & 0 & -0.1528 & 0 & 1.1060 & 0 \end{bmatrix} \qquad (9)$$

The next example is an asymmetric 8-pole filter containing three pure imaginary TZs and four complex TZs. The transfer function of this filter is given as (10). The coefficients of the denominator polynomial have a complex number due to the asymmetric frequency response.

$$t(s)=\frac{1}{\varepsilon}\frac{P_8s^8+P_7s^7+P_6s^6+P_5s^5+P_4s^4+P_3s^3+P_2s^2+P_1s+P_0}{s^8+E_7s^7+E_6s^6+E_5s^5+E_4s^4+E_3s^3+E_2s^2+E_1s+E_0}, \qquad (10)$$

where

E_7=1.7508+j1.2848, E_6=2.3538+j2.0673,
E_5=2.0592+j2.9606, E_4=1.2956+j2.5159,
E_3=0.5732+j1.6588, E_2=0.1569+j0.7259,
E_1=0.0225+j0.2047, E_0=0.0006+j0.0270,
P_8= 0.0008, $P_7=j\,3.9970$, $P_6=-3.8355$, $P_5=j\,4.4737$,
$P_4=-3.5478$, $P_3=-j\,7.9415$, $P_2=9.0129$, $\varepsilon=-j\,242.10$.

The same optimization method is applied to this filter having the coupling scheme as shown in Fig. 4. Because the coefficients

$$
M=\begin{bmatrix}
0 & 0.91636 & 0 & -0.037446 & -0.183243 & 0 & 0 & 0 & 0 & -0.000002 \\
0.91636 & 0.072242 & 0.646439 & -0.286849 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.646439 & 0.632598 & 0.362607 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.037446 & -0.286849 & 0.362607 & -0.065002 & 0.51233 & 0 & 0 & 0 & 0 & 0 \\
-0.183243 & 0 & 0 & 0.512333 & 0.068755 & 0.488674 & 0 & 0 & 0 & -0.04504 \\
0 & 0 & 0 & 0 & 0.488674 & 0.086554 & 0.477464 & 0 & 0.038949 & -0.013941 \\
0 & 0 & 0 & 0 & 0 & 0.477464 & 0.057249 & 0.495301 & -0.148886 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.495301 & 0.326158 & 0.715264 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.038949 & -0.148886 & 0.715264 & 0.106246 & 0.934828 \\
-0.000002 & 0 & 0 & 0 & -0.04504 & -0.013941 & 0 & 0 & 0.934828 & 0
\end{bmatrix} \quad (11)
$$

of the denominator polynomial have both real and imaginary numbers, 24 non-linear equations can be obtained for the goal function. Figure 5 shows the frequency response of transfer function (10) and the coupling matrix (M) given by (11).

Note that the response of the transfer function and that of the coupling matrix cannot be distinguished in the two examples. This agreement verifies the presented synthesis method for the presented topology of the resonator filter.

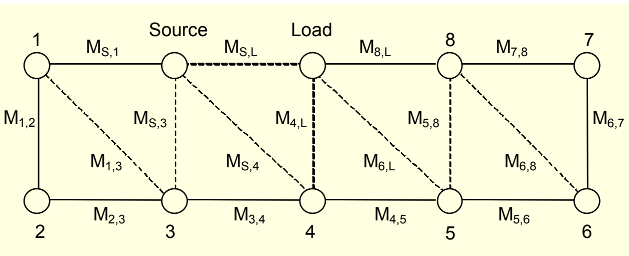

Fig. 4. Coupling scheme of an 8-pole filter with seven TZs.

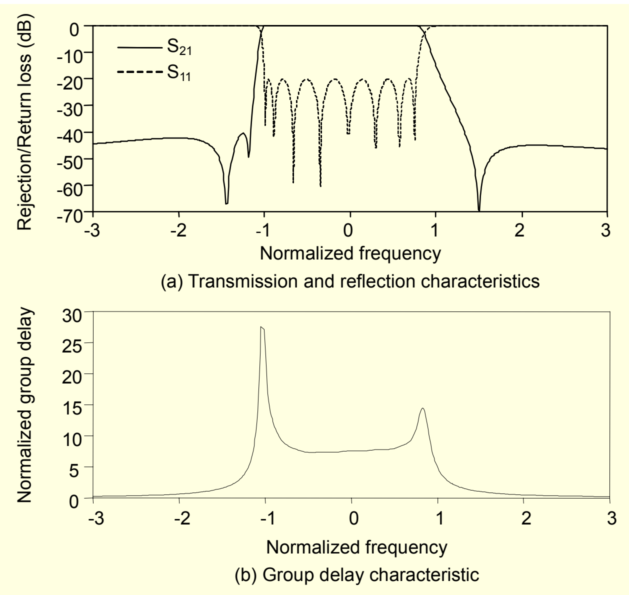

Fig. 5. 8-pole filter with seven TZs. The results agree within the plotting accuracy and cannot be distinguished.

IV. Conclusion

This letter has presented the generalized synthesis method for microwave resonator filters. The relation between the coefficients of the transfer function and the coupling matrix has been also given for the straightforward application of this new method. This synthesis method has been applied to the symmetric 4-pole filter with four TZs and the asymmetric 8-pole filter with seven TZs. The frequency responses of the coupling matrices have been shown to agree well with those of the transfer functions.

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