Angular MST-Based Topology Control for Multi-hop Wireless Ad Hoc Networks

Hwangnam Kim, Eun-Chan Park, SungKee Noh, and Sung Back Hong

ABSTRACT— This letter presents an angular minimum spanning tree (AMST) algorithm for topology control in multi-hop wireless ad hoc networks. The AMST algorithm builds up an MST for every angular sector of a given degree around each node to determine optimal transmission power for connecting to its neighbors. We demonstrate that AMST preserves both local and network-wide connectivity. It also improves robustness to link failure and mitigates transmission power waste.

Keywords—Topology control, minimum spanning tree.

I. Introduction

Topology control in wireless networks has been proposed to discover an appropriate set of neighboring nodes by using a proper transmission power instead of the maximum power. Topology control affects the energy consumption since it adjusts the transmission power to be just large enough to connect to neighbors [1]. Also, topology control increases the network capacity by allowing simultaneous transmission with nonintrusive power, thus, mitigating interference [2]. For this reason, topology control should simultaneously deal with network connectivity [3] and power consumption [4]. We propose a localized topology control algorithm for wireless ad hoc networks, called an angular minimum spanning tree (AMST), which preserves both the local and network connectivity and saves total power consumption.

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II. AMST-Based Topology Control

The AMST consists of three stages: collecting neighbor information, discovering the MST, and determining adequate transmission power. We assume that the maximum power is identical, and that every node knows its relative position within the network.

To collect neighbor information, node *u* broadcasts `Hello` messages, consisting of the position and unique identifier *id*(*u*), with the maximum power P_{max}. Then, the corresponding maximum distance d_{max} is determined from a propagation model. With d_{max} , we can present a wireless network with an undirected simple graph *G*(*V, E*), where *V* is the set of nodes and $E=\{(u,v)\,|\,d(u,v)\le d_{max}, u, v\in V\}$. Therefore, node *u* can determine its neighbor set, $N(u)=\{v\in V(G)\,|\,d(u,v)\le d_{max}\}$. Based on the original topology *G*(*V, E*) and the neighbor information *N*(*u*), *u* constructs its local MSTs for the non-overlapping angular sector of the degree *θ*. Here, n_{MST} is the maximum number of MSTs for node *u*, determined as n_{MST}=360°/*θ*, where *θ* is an angle to determine angular sectors. To determine a local MST, we employ PRIM's algorithm with a new weight function [5], which is basically the distance (*d*(*u, v*)) between two nodes *u* and *v*. It also uses the angle *θ*(*u, v*), defined as the angle between the edge (*u, v*) and the x-axis. Let Θ_i denote the angular range $\Theta_i=\{\theta_i\,|\,(i-1)\theta\le\theta_i<i\theta\}$, where $1\le i\le n_{MST}$, which determines the *i*-th unbounded angular sector of *θ* around *u*. Given an edge (*u, v*), the weight function, $\omega_{\theta_i}: E(G)\rightarrow R$, is

$$\omega_{\theta_i}(u,v)=\begin{cases}\infty & \text{if } d(u,v)>d_{max} \text{ or } \theta(u,v)\notin\Theta_i,\\ d(u,v) & \text{otherwise}.\end{cases}$$

As a tie-breaking rule, AMST chooses the node with smaller identity. PRIM's algorithm makes at most n_{MST} MSTs for node u, resulting in the local topology $\bigcup_{i=1}^{n_{\text{MST}}} M_{\Theta_i}(u)\left(V_{\Theta_i}(u), E_{\Theta_i}(u)\right)$, and finally yields the network topology $G_{\text{AMST}}(V_{\text{AMST}}, E_{\text{AMST}}) = \bigcup_{i=1}^{|V(G)|}\left(\bigcup_{j=1}^{n_{\text{MST}}}\right)M_{\Theta_j}(u_i)\Big)$.

III. Properties of AMST Topology Control

In this section, we prove that AMST preserves neighborhood and network-wide connectivity.

Local connectivity. For any node v_1 and $v_2 \in N(u)$, if there is a path $< w_0 = v_1, w_1, \cdots, w_m = v_2 >$ such that edge $(w_k, w_{k+1}) \in E_{\Theta_i}(u)$, k=0, 1,⋯, m−1, where $w_k \in V_{\Theta_i}(u)$, $1 \le i \le n_{\text{MST}}$, then v_1 is connected to v_2, denoted as $v_1 \to v_2$ in $\bigcup_{i=1}^{n_{\text{MST}}} M_{\Theta_i}(u)$. If $v_1 \to v_2$ and $v_2 \to v_1$, both nodes are connected to each other, which is denoted as $v_1 \leftrightarrow v_2$.

Theorem 1. For any node pair of v_1 and $v_2 \in N(u)$, $v_1 \leftrightarrow v_2$ in $\bigcup_{i=1}^{n_{\text{MST}}} M_{\Theta_i}(u)$.

Proof. If v_1 and $v_2 \in M_{\Theta_i}(u)$, they are definitely connected in $M_{\Theta_i}(u)$ since PRIM's algorithm builds up a connected MST as far as the original graph is connected [5]. Otherwise, let $v_1 \in M_{\Theta_i}(u)$, and $v_2 \in M_{\Theta_j}(u)$, where $1 \le i, j \le n_{\text{MST}}$ and $i \ne j$. $v_1 \leftrightarrow u$ since v_1 is in $V_{\Theta_i}(u)$, and $u \leftrightarrow v_2$ since v_2 is in $V_{\Theta j}(u)$. Therefore, there is a path from v_1 to v_2 passing through u (or vice versa). □

Network connectivity. For any node u and $v \in V(G)$, node u is said to be connected to node v, denoted as $u \Leftrightarrow v$, if there is a path $< w_0 = u_1, w_1, \cdots, w_m = v >$ such that $(w_j, w_{j+1}) \in E(G)$, $j = 0, 1, \cdots, m-1$. Moreover, if $u \Leftrightarrow v$ and $v \Leftrightarrow w$, $u \Leftrightarrow w$.

Lemma. For any node u and $v \in V(G)$, if $d(u, v) \le d_{\max}$ then $u \Leftrightarrow v$ in G_{AMST}.

Proof. Since $d(u, v) \le d_{\max}$, we know $u \in N(v)$ and $v \in N(u)$. Theorem 1 states that $u \leftrightarrow v$ in $\bigcup_{i=1}^{n_{\text{MST}}} M_{\Theta_i}(u)$ or $\bigcup_{i=1}^{n_{\text{MST}}} M_{\Theta_i}(v)$; therefore, $u \Leftrightarrow v$ in G_{AMST}. □

Theorem 2. The topology of G_{AMST} (V_{AMST}, E_{AMST}) preserves network connectivity in the original topology $G(V, E)$ as far as $G(V, E)$ is connected.

Proof. Suppose $G(V, E)$ is connected with $d_{\max}$. Then, there is a path for two nodes u and $v \in V(G)$. Let p denote the path $< w_0 = u, w_1, \cdots, w_m = v >$ from u to v, where each of $(w_i, w_{i+1}) \in E(G)$, for i = 0, 1, ⋯, $m-1$. Since $d(w_i, w_{i+1}) \le d_{\max}$, $w_i \Leftrightarrow w_{i+1}$ in G_{AMST} by the previous lemma. Therefore, an edge (w_i, w_{i+1}) of path p in G is replaced by a path in G_{AMST}, and so path p clearly exists in G_{AMST}. Consequently, G_{AMST} preserves the network connectivity of G. □

IV. Validation

We consider wireless networks consisting of N (= 50) nodes randomly scattered over the area of 1 km×1 km. To determine $P_{\max}$, we assume a two-ray propagation model and 802.11 Wi-Fi devices. Since the minimum receiver power is -80 dBm and $d_{\max}$ is set to 300 m, $P_{\max}$ is about 80 mW.

Figure 1 compares the original topology with the maximum transmission power, the global MST topology, and the topology derived by AMST with n_{MST}= 2. Compared to the original topology, AMST reduces the average node degree significantly. Thus, AMST can avoid unnecessary interference and cut down packet collision probability. Unlike global MST, which builds a unique path between a source and a destination, AMST provides

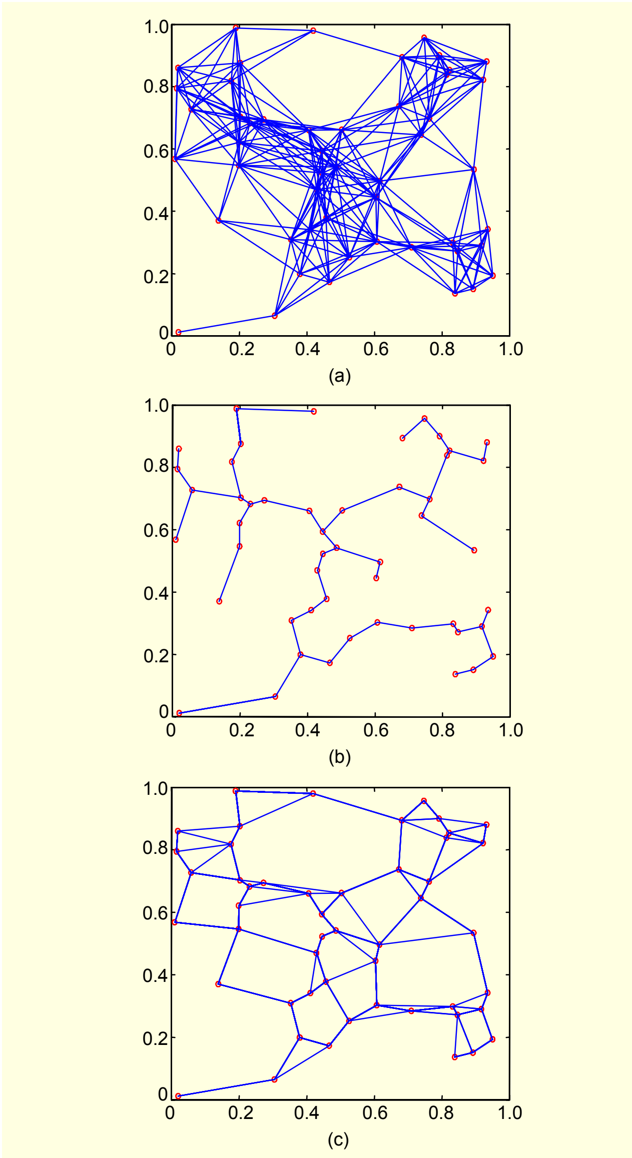

Fig. 1. Topologies derived with (a) the maximum power, (b) global MST, and (c) AMST.

Table 1. Node degree (N_d), path length (P_l), and power consumption ratio (P_C).

Topology	N_d	P_l	P_C
Original	11.5220	2.6960	1.000
Global MST	1.9580	10.4955	0.1237
AMST	3.5460	8.4181	0.1064
CBTC	2.5420	9.0203	0.1459
LMST	2.6740	8.5980	0.1070

Table 2. Effect of localization in building MST.

n_{MST}	N_d	P_l	P_C
Original	11.2460	2.6351	1.0000
1	2.6480	8.3758	0.1088
2	3.4980	8.2759	0.1086
3	3.9260	8.2243	0.1085
4	4.6620	8.1963	0.1084
5	4.9560	8.1963	0.1084
6	5.8320	8.1963	0.1084

redundant paths for most pairs of a source and a destination; thus, the resulting topology becomes more robust to link failures. Moreover, the increased node degree of AMST can decrease path length, which contributes to decreasing transmission power. Table 1 lists and compares three metrics for five topologies, including CBTC [4] (the angle is 180°) and LMST [2]. Compared to the other topologies except the original one, AMST increases the node degree up to 75% and decreases the path length up to 20%. Moreover, the power consumption in AMST is remarkably lower than that in the original topology; the gain of power consumption is about 10. Compared to global MST and CBTC, AMST reduces power consumption by about 14% and 27%, respectively. Table 2 presents the effect of n_{MST} on the performance of AMST. As n_{MST} increases from 1 to 6, the node degree increases from 3.65 to 5.83; however, the path length and power consumption were gradually improved until n_{MST}=4. Based on the table, we recommend setting n_{MST} to 2 or 3 under the rationale that the complexity increases in proportion to n_{MST}.

Figure 2 compares the local connectivity from the perspective of a node V when we use LMST, CBTC, and AMST. In the network, 5 nodes are placed in an area of 1 km×1 km. We set the maximum distance to 1 km, and set the angle θ for both CBTC and AMST to 90°. Figure 2 shows that LMST (dotted line) produces an MST-rooted *V*, consisting of *X*, *Y*, *W*, and *Z*. The MST may incur bandwidth and power wastage due to multiple transmissions for one effective delivery. On the other hand, CBTC (dashed line), generates a partial tree

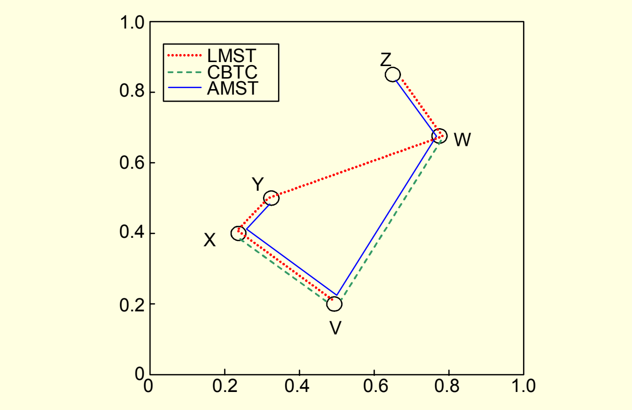

Fig. 2. Local topologies derived with LMST, CBTC, and AMST.

rooted at *V*. This is because CBTC gradually increases *V*'s transmission power and immediately stops when it finds *X* and *W* in the left-upper and right-upper sectors of 90°. The solid line in Fig. 2 shows that AMST constructs two MSTs including all neighbors but produces better local connectivity than LMST. For example, AMST allows *V* to reach *Z* with two hops while LMST requires 4 hops. From the perspective of local connectivity, AMST and LMST are better than CBTC, as they provide local information of connecting to its original neighbors; however, AMST can produce a shorter path to a neighbor than LMST since its angular property imposes directional constraint on the topology.

V. Conclusion

We proposed the AMST algorithm for topology control in multi-hop wireless ad hoc networks. AMST can save transmission power while preserving both local and network connectivity.

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