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THEORY

How Much Benefit Can Multipacket Reception Channel Bring to CSMA?

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ABSTRACT This paper investigates the critical issue of maintaining system stability for multipacket reception (MPR) p-persistent carrier sense multiple access (CSMA) systems. When multiple users with individual queues simultaneously transmit a packet to an access point (AP) via CSMA, the number of successful transmissions and their identities are determined probabilistically in the MPR channel. Stability signifies that none of the users' queues grows unbounded. The stability region is a significant measure, addressing all potential permutations of mean packet arrival rates for the users to maintain bounded queue lengths. The work begins by considering a system with two users characterized by differing mean packet arrival rates and (re)transmission probabilities. Subsequently, it examines an N-user system in which each user shares an identical packet arrival rate and retransmission probability. A backoff algorithm is then proposed for these N users to utilize in order to stabilize their queues. The paper concludes with numerical studies illustrating the stability region as a function of user parameters, demonstrating how the proposed backoff algorithm can be used to maximize throughput.

INDEX TERMS Stability region, multipacket reception (MPR), CSMA, IEEE 802.11.

I. INTRODUCTION

Demands on bandwidth, fuelled by social network services (SNSs), streaming video services, YouTube, online gaming, etc., have explosively grown in the past few decades. IEEE 802.11 wireless local area networks (WLANs) have met those demands in home, business, and public spaces while consuming large amounts of data via wireless cellular networks has still been costly. The proliferation of WLAN-enabled smartphones and tablet PCs has enabled users to enjoy the internet anywhere easily.

In order to fulfill ever-increasing demands, IEEE 802.11 has evolved continually as IEEE 802.11a/b/g/n/ac/ax. For instance, the maximum data rate has been increased from 11 Mbps to 54 Mbps and further up to 6.9 Gbps with the introduction of downlink multi-user multi-input multi-output (MU-MIMO) antenna and a higher high-order modulation schemes in the physical layer [1]. In the medium access control (MAC) layer, the random access protocol of IEEE 802. 11 has still been based on carrier sense multiple access (CSMA) protocol even as uplink traffic has kept increasing due to SNSs, video conferencing, online lectures, etc. So far, CSMA protocol works with a single packet reception (SPR) channel; that is, none of them make a successful transmission if more than one packets are (re)transmitted at the same time. To cope with increasing uplink traffic, IEEE 802.11ax introduces uplink MU-MIMO so that upon simultaneous multiple packet transmissions,

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some of them can be successfully decoded, which is called multipacket reception (MPR) channel [2], [3] in the literature. Prior to 802.11ax, various MPR channels and MAC protocols to support them have been proposed for CSMA system [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [24], [28], [29], [30], [31], [32], [33].

In the random access systems either slotted ALOHA [3], [4], [5] or CSMA [6], [7], [8], where users have a queue to buffer incoming packets, the system is said to be unstable, if at least one user's queue grows unbounded. It is thus an essential question of how much an MPR channel can affect the stability of users' queueing process. It is also important to control users' retransmissions so that each user's queue grows finite for a given set of packet arrival rates of the users in the system. To put it in another way, we can ask what the set of the maximum allowable packet arrival rates of the users can be so that a retransmission control scheme can exist in order to stabilize the system. As the importance of MPR capability for CSMA has risen, this work explores these questions for CSMA systems with a generic MPR channel.

A. RELATED WORKS

Studies on MPR CSMA systems in the literature can be classified into two groups such as synchronous [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [24], and asynchronous [28], [29], [30], [31], [32], [33]: In both systems, the time is divided into sensing slots, when the channel is idle. The users are synchronized at each sensing slot and the packet transmission occurs just at the end of an idle slot after the channel is sensed idle. In particular, in synchronous systems, the packet transmissions are aligned with slot boundary. However, in asynchronous systems, other packet transmissions are assumed to be allowed even during the ongoing packet transmissions.

This work focuses on the synchronous systems since implementing the synchronous system might be relatively easier than the asynchronous ones and more feasible in the physical layer. In [9], under the assumption that the users can hold only a packet, the maximum allowable packet arrival rate is examined for the number of backlogged users to grow finite in MPR CSMA. In such a case, a packet and a user are not distinguishable. In contrast, our work assumes that the number of users is finite, but they have a queue of infinite length to hold incoming packets. In [10], [11], and [12] it is examined how MIMO can materialize MPR CSMA. In [13], it is proposed that the users in CSMA exploit their channel state; that is, upon idle channel, the users can access when their signal-to-noise ratio (SNR) or channel state is above a threshold. In [14], access fairness is examined for MPR CSMA systems implemented by MIMO. Let us define M-MPR channel that the users make a successful transmission if the number of transmitting users is less than or equal to M. In [15], various properties of CSMA systems with M-MPR channel have been examined. For example, throughput increases superlinearly as M increases. Under the assumption of Poisson traffic with mean aggregate rate G, an analytical throughput model for slotted non-persistent CSMA is examined [16]. Some queueing model of $M/G/1$ with vacations is applied to the users' queue in the MPR CSMA system in [17] such that the condition for bounded access delay is analyzed. Backoff algorithms are developed to maximize the system throughput [18], [19]. New uplink and downlink protocols for MU-MIMO CSMA are proposed in [20].

The performance of CSMA systems has been extensively investigated using the mean field approximation (MFA) in [21], [22], [23], and [24]. When all the users always have a packet to send, i.e., in saturated condition, and the number of users is large, the MFA can convert a discrete-time and discrete-state Markov process of user backoff procedures in IEEE 802.11, i.e., distributed coordination function (DCF), into a set of ordinary differential equations (ODEs). The solution of the ODEs is obtained as the form of a fixed point from which the system's performance can be estimated. The MFA is often juxtaposed with Bianchi's analysis [21], [22], where the decoupling assumption has been made; that is, the backoff processes of the users' DCF become independent or decoupled when all the users are saturated. Especially, [22] used MFA to analyze the validity of Bianchi's analysis by determining the stability of the ODEs and providing a stability condition. In [24], the stability region of CSMA system with an MPR channel is examined with MFA. In particular, multiple classes of users are considered: The users in the same class have identical packet arrival rates and transmission probabilities but not in different classes.

Compared to [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], and [20], this work investigates the stability region of MPR CSMA systems with M-MPR capability when the users have a queue of unlimited length. To do this, we use the stochastic dominant systems to characterize the stability region. Its advantage over the MFA is accurately characterizing the systems' stability region with two users having different packet arrival rates and transmission probabilities, called *asymmetric users*, or with three users in an S-ALOHA system [26]. However, for systems with more than three asymmetric users, the method of using stochastic dominant systems suffers from the curse of dimensionality in queueing analysis. While it has been demonstrated that the MFA works well for a large number of (saturated) users, i.e., as the number tends to infinity, it is not rigorously verified that its results can be accurate for a few users. Note that in [27], the stability region of S-ALOHA has been investigated by the MFA. Together with the result in [24], we extend the result of the stability region for two-user into finite users CSMA system and demonstrate that our work can be a good complement to the stability region obtained by the MFA [24].

B. CONTRIBUTIONS AND ORGANIZATION

The contributions of this work are summarized as follows:

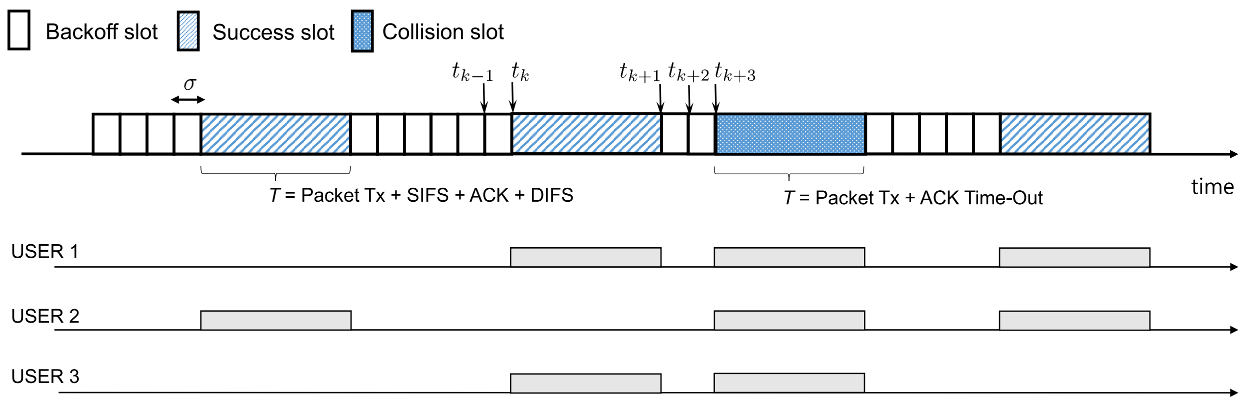


FIGURE 1. **MPR CSMA with MPR capability $M = 2$.**

- For the system with two asymmetric users, conditional and unconditional stability regions are fully characterized. Depending on the MPR channel and users' parameters, it is shown how the stability region of the system can change, e.g., when the stability region of MPR CSMA can exceed that of an ideal TDMA system. Fig. 4 in Section III visualizes this as one of the main results of this work.
- The stability condition of the system with N symmetric users is examined. Then, in order to maximize the system throughput under the stability condition, a backoff algorithm is proposed based on Bayesian estimation for the number of backlogged users. The results demonstrate that the performance of the proposed algorithm is quite close to that of the genie-aided algorithm [36] using perfect knowledge of backlog size. Furthermore, under environments with a time-varying number of users and packet arrival rate, it is also illustrated how well the proposed algorithm estimates the actual backlog size.

Throughout this paper, a boldfaced lowercase letter denotes a row vector, e.g., $\mathbf{x} = [x_i]$, while a boldfaced uppercase letter denotes a set. In addition, for probability x, $\overline{x}$ denotes its complement, i.e, $\overline{x} = 1 - x$, whereas for a set $\mathbf{X}$ its complement is denoted by $\overline{\mathbf{X}} = \mathbf{X}^c$. Moreover, $|\mathbf{X}|$ denotes the cardinality of set $\mathbf{X}$.

The organization of this paper is as follows. In Section II, we introduce our system model and the definitions of the stability regions. Section III provides the stability analysis of the systems with two users and N users by using stochastic dominant systems. We discuss numerical studies in Section IV. Finally, we give concluding remarks in Section V.

II. SYSTEM MODEL

A. MPR CSMA SYSTEMS

Suppose a slotted CSMA system in which time is equally divided into a backoff (or sensing) slot of σ μsec, when the channel is idle as shown in Fig. 1. There are a total of N users, each with a buffer of infinite length to store arriving packets. Let us denote the index set of N users by $\mathcal{N} = \{1, 2, \ldots, N\}$. We assume that user $i \in \mathcal{N}$ has packet arrivals according to a Poisson process with a mean rate λ_i (packets/μsec), and the size of each packet is equal. According to p-persistent CSMA, N users can communicate with an AP wirelessly as follows: When sensing the channel idle, user i with a non-empty queue (re)transmits its packet at the head of the queue with probability p_i until the packet is successfully transmitted.

1) A GENERIC MPR CHANNEL

Let us characterize an MPR channel of an AP as follows: Let $\mathcal{T}$ be the index set of the users who make a packet transmission in set $\mathcal{N}$, and $\mathcal{R}$ denotes the index set of the users whose packets are successfully decoded in set $\mathcal{T}$. We have $\mathcal{R} \subseteq \mathcal{T} \subseteq \mathcal{N}$. Let us denote the conditional probability $q_{\mathcal{R}|\mathcal{T}}$ that the users of set $\mathcal{R}$ successfully transmit given that set $\mathcal{T}$ of users transmit. For a two-user system, we specifically define as follows:

- $q_{1|\{1\}}$: user 1's success probability when only user 1 transmits.
- $q_{2|\{2\}}$: user 2's success probability when only user 2 transmits.
- $q_{1,\underline{2}|\{1,2\}}$: the probability that only user 1's transmission is successful when both users transmit.
- $q_{\underline{1},2|\{1,2\}}$: the probability that only user 2's transmission is successful when both users transmit.
- $q_{1,2|\{1,2\}}$: the probability that both users succeed when both users transmit.
- $q_{\underline{1},\underline{2}|\{1,2\}}$: the probability that both users fail when both users transmit.

When two users transmit simultaneously, we have

$$q_{1,\underline{2}|\{1,2\}} + q_{\underline{1},2|\{1,2\}} + q_{1,2|\{1,2\}} + q_{\underline{1},\underline{2}|\{1,2\}} = 1. \quad (1)$$

The probability $q_{\mathcal{R}|\mathcal{T}}$ for this two-user system can be expressed as

$$q_{1|\{1,2\}} = q_{1,\underline{2}|\{1,2\}} + q_{1,2|\{1,2\}} \quad \text{and} \quad (2)$$

$$q_{2|\{1,2\}} = q_{\underline{1},2|\{1,2\}} + q_{1,2|\{1,2\}}. \quad (3)$$

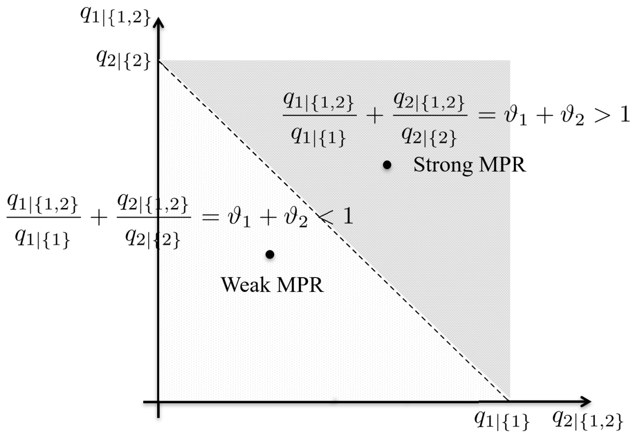


FIGURE 2. **Strong and weak MPR channels.**

Thus, $q_{i|\{1,2\}}$ is the probability that user i makes a successful transmission, whether the other user succeeds or not.

Let us define ϑ_1 and ϑ_2 as

$$\vartheta_1 \triangleq \frac{q_{1|\{1,2\}}}{q_{1|\{1\}}}, \quad \text{and} \quad \vartheta_2 \triangleq \frac{q_{2|\{1,2\}}}{q_{2|\{2\}}}. \tag{4}$$

When $q_{i|\{i\}} < 1$, it can represent a noisy channel. The MPR-capable system is said to be *strong* or *weak*, if we have

$$\text{MPR} = \begin{cases} \text{Strong}, & \text{for } \vartheta_1 + \vartheta_2 > 1, \\ \text{Critical}, & \text{for } \vartheta_1 + \vartheta_2 = 1, \\ \text{Weak}, & \text{for } \vartheta_1 + \vartheta_2 < 1, \end{cases} \tag{5}$$

which is depicted in in Fig. 2. Assuming that $q_{i|\{i\}} = 1$, and using $q_{1,\underline{2}|\{1,2\}} + q_{\underline{1},2|\{1,2\}} = 1 - q_{1,2|\{1,2\}} - q_{\underline{1},\underline{2}|\{1,2\}}$ in (1) and (2), we can find an alternative characterization of the strong MPR channel, i.e., $\vartheta_1 + \vartheta_2 > 1$ as

$$q_{1,2|\{1,2\}} > q_{\underline{1},\underline{2}|\{1,2\}}. \tag{6}$$

This implies that when both users transmit, the channel is a strong MPR if the probability of both successes is greater than that of both failures. For the critical and weak MPR, we have $q_{1,2|\{1,2\}} = q_{\underline{1},\underline{2}|\{1,2\}}$, and $q_{1,2|\{1,2\}} < q_{\underline{1},\underline{2}|\{1,2\}}$, respectively.

Let us return to the MPR channel in Fig. 1, where three users compete for channel access. In this system, we assume that if more than two packets are transmitted simultaneously, e.g., at instant t_{k+3}, a collision occurs; that is, no one makes a successful transmission. Otherwise, the users' transmission becomes successful. In this case, we have conditional probabilities $q_{1|\{1\}} = q_{2|\{2\}} = q_{3|\{3\}} = q_{1,2|\{1,2\}} = q_{1,3|\{1,3\}} = q_{2,3|\{2,3\}} = q_{\underline{1},\underline{2},\underline{3}|\{1,2,3\}} = 1$, whereas other conditional probabilities are zero.

We additionally assume that the successful packet transmission period equals the collision period, denoted by T in Fig. 1. This corresponds to the basic access of IEEE 802.11. We can relate T upon success to IEEE 802.11 as follows:

$$T = \text{Packet TX} + \text{SIFS} + \text{ACKTime} + \text{DIFS}. \tag{7}$$

Similarly, upon collision, we have

$$T = \text{Packet TX} + \text{ACK Time-out}. \tag{8}$$

Note that Packet TX, SIFS, ACKTime, ACK Time-out, and DIFS denote the duration of a packet transmission time, that of short interframe space (SIFS), that of acknowledgment (ACK), that of ACK time-out, and that of distributed coordinate function interframe space (DIFS), respectively.

2) REALIZATION OF MPR CHANNEL WITH MIMO

We discuss how the strong and weak MPR channels can be formed by investigating the MU-MIMO, one of the typical transmission techniques to realize MPR. Suppose one AP has two receive antennas and two users, each with one transmit antenna. Let the channel between each transmit and receive antenna pair be Rayleigh fading. Then, we have the following two cases:

- If both the users transmit simultaneously, the received SNR for user i (for $i = 1, 2$) at the AP has an exponential distribution with mean μ_i when zero-forcing MIMO decoding was performed [12]. Consequently, user i's success probability can be written as

$$q_{i|\{1,2\}} = \exp\left(-\frac{\gamma_i}{\mu_i}\right), \tag{9}$$

  where γ_i is user i's SNR threshold for successful decoding at the AP.
- If only user i transmits, the received SNR at the AP has Gamma distribution with the shape parameter value 2, as described in [12]. Consequently, user i's success probability can be written as

$$q_{i|\{i\}} = \left(1 + \frac{\gamma_i}{\mu_i}\right) \cdot \exp\left(-\frac{\gamma_i}{\mu_i}\right). \tag{10}$$

Now, we can obtain the following.

$$\vartheta_1 + \vartheta_2 = \frac{\mu_1}{\gamma_1 + \mu_1} + \frac{\mu_2}{\gamma_2 + \mu_2}. \tag{11}$$

As can be seen, depending on the choice of the threshold of γ_i, the MPR channel can be strong or weak. For example, if $\gamma_i > \mu_i$ for $i = 1, 2$, it is weak MPR channel.

It is notable that (9) is obtained by assuming that the AP is able to estimate the two users' channel gains perfectly, even if they randomly transmit. If the simultaneously transmitting users transmit pilot signals at the same resource, the AP cannot estimate their channel gains, which may result in more frequent decoding errors. If we denote such performance loss by $a(\leq 1)$ for simplicity, (9) can be rewritten as $q_{i|\{1,2\}} = a \cdot \exp\left(-\frac{\gamma_i}{\mu_i}\right)$. Since $a \leq 1$, the weak MPR channel may occur frequently in practice.

3) NON-ORTHOGONAL MULTIPLE ACCESS (NOMA) FOR MPR

Another possible transmission technique that can realize the MPR channel is the non-orthogonal multiple access (NOMA) in the power domain. The users transmit their packet by choosing one of two *receive* power levels P_1 and $P_2 (< P_1)$ with transmit power control. For NOMA random access, each user can select one power level with probability $\frac{1}{2}$ [37], [38]. Only when two transmitting users select different receive power levels each, which occurs with probability $\frac{1}{2}$, the AP can decode both users' packets by applying successive interference cancellation (SIC). If the two users choose the same power level, both users fail. Consequently, we have $q_{i|1,2} = \frac{1}{2}$ for $i = 1, 2$. For $q_{i|i} = 1$, we have

$$\vartheta_1 + \vartheta_2 = 1, \tag{12}$$

which is a critical MPR channel.

B. QUEUEING PROCESS

Since the length of idle slot (σ) and that of a transmission period such as a successful packet transmission slot and a collision T are different, we define an embedded point in this CSMA system as the end of a time epoch, where either idleness or transmission is over. Let t_k denote the time epoch of an embedded point and $Q_i(t_k)$ denote the queue length process of the i^{th} user at the corresponding time epoch as in Fig. 1. Then, the evolution of $Q_i(t_k)$ over time can be expressed as

$$Q_i\left(t_{k+1}\right) = \left[Q_i\left(t_k\right) - C_i\left(t_k\right)\right]^+ + \lambda_i(t_{k+1} - t_k), \tag{13}$$

where $[x]^+ = \max\{x, 0\}$ and $\lambda_i(t_{k+1} - t_k)$ denotes the number of packet arrivals at the i^{th} user between t_k and t_{k+1}. Particularly, the service process, $C_i\left(t_k\right)$, is expressed as

$$C_i(t_k) = \begin{cases} 1, & \text{with probability } \ p_{s,i}, \\ 0, & \text{with probability } \ \overline{p}_{s,i}, \end{cases} \tag{14}$$

where $p_{s,i}$ denotes the packet transmission success probability via random access.

We consider two cases for $p_{s,i}$ as follows: The *first* case is a two-user MPR CSMA system: If two users have at least one packet to send, $p_{s,i}$ can be expressed as

$$p_{s,i} = p_i \left(q_{i|\{i\}}\overline{p}_{3-i} + q_{i|\{i,3-i\}}p_{3-i}\right). \tag{15}$$

Note that for $i = 1$ (or 2), we have $3 - i = 2$ (or 1) in (15), which indicates the other user.

The *second* case is an N-user system: In this case, $p_{s,i}$ depends on the physical layer. Let us assume an MPR channel that if the number of simultaneously transmitting users is not more than M, all of them make a successful transmission. When N users have non-empty queue, user i's transmission is successful if

$$p_{s,i} = p_i \sum_{\substack{\mathcal{T} \subseteq \mathcal{N}\setminus i, \\ |\mathcal{T}| \le M-1}} \prod_{j \in \mathcal{T}} p_j \prod_{k \in \mathcal{T}^c \setminus i} \overline{p}_k, \tag{16}$$

where $\mathcal{N}\setminus i$ is the set $\mathcal{N}$ except user i.

Let $\mathbf{Q}(t_k) = [Q_i(t_k)]$ for $i \in \{1, \ldots, N\}$ denote the entire queueing process of N users in slotted CSMA systems. According to (13), $\mathbf{Q}(t_k)$ is an N-dimensional Markov process. Then, the queueing process (or system) $\mathbf{Q}(t_k)$ is said to be stable if the following holds:

$$\lim_{t_k \to \infty} \Pr\left[\mathbf{Q}(t_k) < \mathbf{y}\right] \equiv F\left(\mathbf{y}\right) \text{ and } \lim_{\mathbf{y} \to \infty} F\left(\mathbf{y}\right) = 1. \tag{17}$$

Because the existence of the limiting distribution as shown in (17) implies the positive recurrence of the aperiodic and irreducible Markov chain corresponding to $\mathbf{Q}(t_k)$, it implies that, e.g., for a stable system with two users, we must have $\Pr[Q_1(t_k) = 0, Q_2(t_k) = 0] > 0$, $\Pr[Q_1(t_k) = 0, Q_2(t_k) > 0] > 0$ and $\Pr[Q_1(t_k) > 0, Q_2(t_k) = 0] > 0$ as t_k goes to infinity. This means that none of the two users can be saturated.

C. ANALYTICAL FRAMEWORK OF STABILITY REGION

This work makes use of the analytical framework in [6]: First, we denote by $\boldsymbol{\lambda} = [\lambda_i]$ and $\mathbf{p} = [p_i]$ for $i \in \{1, 2, \ldots, N\}$ the packet arrival rate vector and the (re)transmission probability vector of the system with N users, in which λ_i and p_i are the mean packet arrival rate and the (re)transmission probability of user i. The conditional and unconditional stability regions can be defined as follows.

Definition 1. *The conditional stability region subject to the (re)transmission probability vector* $\mathbf{p}$ *is defined as a vector set of having all possible combinations of* $\boldsymbol{\lambda}$*, denoted by* $\boldsymbol{\Lambda}_{\mathbf{p}}$*, as long as (17) holds for a given* $\mathbf{p}$.

Definition 2. *The (unconditional) stability region, denoted by* $\boldsymbol{\Lambda}$*, is defined as a vector set with all possible combinations of* $\boldsymbol{\lambda}$ *that satisfies (17) achieved by some vector* $\mathbf{p}$.

Unless otherwise specified, this paper refers to the stability region as the unconditional one. Note that the difference between $\boldsymbol{\Lambda}_{\mathbf{p}}$ and $\boldsymbol{\Lambda}$ is that we can obtain $\boldsymbol{\Lambda}_{\mathbf{p}}$ for a given $\mathbf{p}$, whereas we have $\boldsymbol{\Lambda}$ if there exists some $\mathbf{p}$ of making $\boldsymbol{\Lambda}$ feasible. Accordingly, we can express $\boldsymbol{\Lambda}$ as

$$\boldsymbol{\Lambda} = \bigcup_{\mathbf{p} \in [0,1]^N} \boldsymbol{\Lambda}_{\mathbf{p}}, \tag{18}$$

where the vector $\mathbf{p} \in [0, 1]^N$ has its i^{th} element $p_i \in [0, 1]$ for $i \in \{1, 2, \ldots, N\}$.

Suppose that system $\mathcal{S}$ has two users, say user 1 and user 2. Let Q_1 (or Q_2) denote user 1's (or user 2's) queue length. Since $C_i(t_k)$ for $i = 1, 2$ in (13) depends on each user's queue state, i.e., empty or not, it is not easy to obtain the conditional stability region, $\boldsymbol{\Lambda}_{\mathbf{p}}$, by using a two-dimensional Markov chain for the system with two users.

In order to facilitate our stability analysis on this *original* system $\mathcal{S}$, we introduce a *stochastic dominant* system as a hypothetical auxiliary system [4]; we shall see that it is relatively easier to obtain the conditional stability region of the (stochastic) dominant systems, which is denoted by $\boldsymbol{\Lambda}^*_{\mathbf{p}}$, because it enables us to circumvent the analysis of the two-dimensional Markov chain.

Let $\mathcal{S}^*_1$ (or $\mathcal{S}^*_2$) denote a dominant system, in which user 1 (or user 2) is designated as a dominant user in the system. The role of the dominant user 1 (or user 2) in $\mathcal{S}^*_1$ (or $\mathcal{S}^*_2$) is to continue to transmit *dummy* packets with probability p_1 (or p_2) even if Q_1 (or Q_2) $= 0$, while it transmits the *real* packet at the head of its queue if Q_1 (or Q_2) > 0. Note that the dummy packet also occupies the transmission time and can collide with the other user's dummy or real packet transmissions. In addition to $\mathcal{S}_1^*$ and $\mathcal{S}_2^*$, we can have another dominant system denoted by $\mathcal{S}_0^*$, in which both users continue to transmit dummy packets when their queues are empty.

The physical difference between the original system $\mathcal{S}$ and the dominant system $\mathcal{S}_i^*$ is that the packet transmission of dominant user i can collide with that of the other non-dominant user 1 more often in the dominant system than original system $\mathcal{S}$, because dominant user i transmits either real packets if $Q_i > 0$, or dummy packets when $Q_i = 0$. Thus, Q_1 and Q_2 in the dominant systems would be at least larger than those in $\mathcal{S}$ at the embedded points if the queueing process starts with the users having identical initial conditions on their queues in both systems. More details are found in [6]. The dominant system can also make it simple to analyze the queueing process of the non-dominant user because the state of the queue of the dominant user is always non-empty. More specifically, as we do not need to consider the state of the dominant user's queue, the queueing analysis for the non-dominant user is reduced to an $M/G/1$ queueing analysis.

To obtain the conditional stability region of a dominant system, let $\mathbf{\Lambda}_{\mathbf{p},i}^*$ denote the conditional stability region of the corresponding dominant system, $\mathcal{S}_i^*$ for $i = 0, 1, 2$, whose definition follows that of $\mathbf{\Lambda}_{\mathbf{p}}$. The overall conditional stability region $\mathbf{\Lambda}_{\mathbf{p}}^*$ of the dominant systems is the union of the stability regions of each dominant system:

$$\mathbf{\Lambda}_{\mathbf{p}}^* = \bigcup_{i=0}^{2} \mathbf{\Lambda}_{\mathbf{p},i}^*. \tag{19}$$

In comparison with the original system, we can have

$$\mathbf{\Lambda}_{\mathbf{p}} \supseteq \mathbf{\Lambda}_{\mathbf{p}}^*. \tag{20}$$

This implies that the conditional stability region of the dominant systems is a subset or an identical set of the original system for the mean packet arrival rates due to dummy packet transmissions. After obtaining $\mathbf{\Lambda}_{\mathbf{p}}^*$, we can find $\mathbf{\Lambda}_{\mathbf{p}}$ by utilizing the *indistinguishableness* property between the original system $\mathcal{S}$ and the dominant systems [4], [6]: As the mean packet arrival rate λ_i increases in the dominant systems so that the dummy packets are completely replaced with the real packets, the original and stochastic dominant systems are not distinguishable, i.e., identical. At the boundary, if the dominant system is unstable, then the original system is also unstable. Thus, the equality holds in (20). In what follows, we obtain $\mathbf{\Lambda}_{\mathbf{p},i}^*$ for $i = 0, 1, 2$, respectively.

III. STABILITY OF MPR CSMA SYSTEMS

Let us consider $\mathcal{S}_0^*$, where both users are designated as dominant users, i.e., both users keep on transmitting real or dummy packets. Therefore, in calculating one user's packet transmission success probability, denoted by $p_{s,i}$ for $i = 1, 2$, it is unnecessary to take into account the other user's queue state, i.e., empty or non-empty. Thus, we can write $p_{s,i}$ as in (15).

By applying Loynes' theorem [34], which shows that a queueing system is stable only if the mean packet arrival rate to user i is less than its service rate, we have

$$L\lambda_i < p_{s,i} \quad \text{for} \quad i = 1, 2, \tag{21}$$

where L denotes the average time between two consecutive embedded points, i.e., $L = \mathbb{E}[t_{k+1} - t_k]$. We can obtain L as

$$L = \overline{p}_1\overline{p}_2\sigma + (1 - \overline{p}_1\overline{p}_2)T, \tag{22}$$

which is also independent of both users' queue states. The left-hand side (LHS) of (21) indicates the average number of arriving packets to user i during L, while the right-hand side (RHS) represents the average number of packets successfully transmitted. Using (21), the following lemma finds the conditional stability region of the dominant system $\mathcal{S}_0^*$.

Lemma 1. *The conditional stability region of stochastic dominant system $\mathcal{S}_0^*$ in (19) is expressed as*

$$\mathbf{\Lambda}_{\mathbf{p},0}^* = \{(\lambda_1, \lambda_2) | \lambda_1 < \mathcal{A}_{\mathbf{p}}, \lambda_2 < \mathcal{B}_{\mathbf{p}}\}, \tag{23}$$

where $\mathcal{A}_{\mathbf{p}}$ and $\mathcal{B}_{\mathbf{p}}$ are given as

$$\mathcal{A}_{\mathbf{p}} = \frac{q_{1|\{1\}}}{T} \frac{p_1(1 - \overline{\vartheta}_1 p_2)}{1 - \overline{c}\,\overline{p}_1\overline{p}_2} \tag{24}$$

and

$$\mathcal{B}_{\mathbf{p}} = \frac{q_{2|\{2\}}}{T} \frac{p_2(1 - \overline{\vartheta}_2 p_1)}{1 - \overline{c}\,\overline{p}_1\overline{p}_2}. \tag{25}$$

The variable c is defined as $c \triangleq \frac{\sigma}{T}$.

Proof: See Appendix A. ■

The following lemma investigates the conditional stability region of the remaining dominant systems.

Lemma 2. *The conditional stability regions of stochastic dominant systems $\mathcal{S}_1^*$ and $\mathcal{S}_2^*$ in (19) are expressed respectively as*

$$\mathbf{\Lambda}_{\mathbf{p},1}^* = \{(\lambda_1, \lambda_2) | \lambda_1 < \alpha_{p_1}(\lambda_2), \lambda_2 < \mathcal{B}_{\mathbf{p}}\}, \tag{26}$$

and

$$\mathbf{\Lambda}_{\mathbf{p},2}^* = \{(\lambda_1, \lambda_2) | \lambda_1 < \mathcal{A}_{\mathbf{p}}, \lambda_2 < \beta_{p_2}(\lambda_1)\}. \tag{27}$$

In (26) and (27), $\alpha_{p_1}(\lambda_2)$ and $\beta_{p_2}(\lambda_1)$ are respectively expressed as

$$\alpha_{p_1}(\lambda_2) = \frac{q_{1|\{1\}}p_1}{T} \left(\frac{1}{1 - \overline{p}_1\overline{c}} + \left(\vartheta_1 - \frac{1}{1 - \overline{p}_1\overline{c}} \right) \frac{\frac{\lambda_2 T}{q_{2|\{2\}}}}{1 - \overline{\vartheta}_2 p_1} \right) \tag{28}$$

and

$$\beta_{p_2}(\lambda_1) = \frac{q_{2|\{2\}}p_2}{T} \left(\frac{1}{1 - \overline{p}_2\overline{c}} + \left(\vartheta_2 - \frac{1}{1 - \overline{p}_2\overline{c}} \right) \frac{\frac{\lambda_1 T}{q_{1|\{1\}}}}{1 - \overline{\vartheta}_1 p_2} \right). \tag{29}$$

Proof: See Appendix B. ■

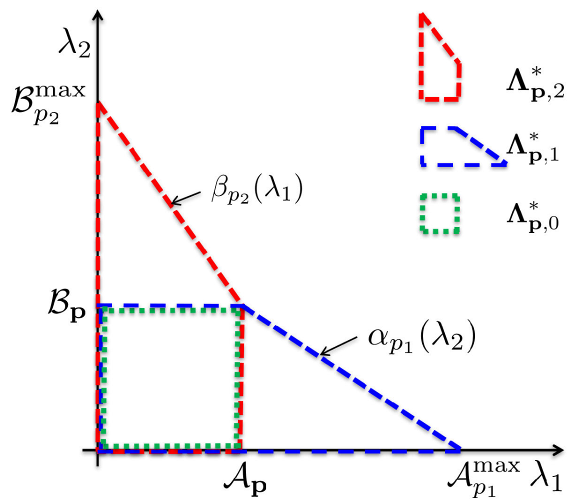


FIGURE 3. **Conditional stability region of a two-user *p*-persistent CSMA system.**

The region of $\boldsymbol{\Lambda}^*_{\mathbf{p},0}$ is depicted as a small rectangle in Fig. 3, where the region of $\boldsymbol{\Lambda}^*_{\mathbf{p},2}$ is also shown. Note that $\beta_{p_2}(\lambda_1)$ is the service rate of the user 2's queue. When $\lambda_1 = 0$ in (29), or $\pi_0 = 1$, $\beta_{p_2}(\lambda_1)$ can be

$$\beta_{p_2}(\lambda_1) \leq \frac{p_2 q_{2|\{2\}}}{\overline{p}_2\sigma + p_2 T} = \frac{q_{2|\{2\}}}{T} \cdot \frac{p_2}{1-\overline{p}_2\overline{c}} = \mathcal{B}_{p_2}^{\max}, \quad (30)$$

in which $\mathcal{B}_{p_2}^{\max}$ is the service rate of user 2's queue when user 1 always has nothing to send.

Similarly, the upper-bound of $\alpha_{p_1}(\lambda_2)$ is given by

$$\alpha_{p_1}(\lambda_2) \leq \frac{p_1 q_{1|\{1\}}}{\overline{p}_1\sigma + p_1 T} = \frac{q_{1|\{1\}}}{T} \cdot \frac{p_1}{1-\overline{p}_1\overline{c}} = \mathcal{A}_{p_1}^{\max}, \quad (31)$$

where the equality holds when the probability of user 2's queue length is zero, i.e., $\phi_0 = 1$ due to $\lambda_2 = 0$. This is given in (57) in Appendix C. As illustrated in Fig. 3, it is easy to check that $\boldsymbol{\Lambda}^*_{\mathbf{p},0} \subseteq \boldsymbol{\Lambda}^*_{\mathbf{p},1}$ and $\boldsymbol{\Lambda}^*_{\mathbf{p},0} \subseteq \boldsymbol{\Lambda}^*_{\mathbf{p},2}$. Therefore, we have $\boldsymbol{\Lambda}^*_{\mathbf{p}} = \boldsymbol{\Lambda}^*_{\mathbf{p},1} \bigcup \boldsymbol{\Lambda}^*_{\mathbf{p},2}$.

Since we obtained the conditional stability region of the dominant systems thus far, let us move to that of the original system.

Theorem 1. *Given retransmission probability vector* $\mathbf{p}$*, the conditional stability region of a two-user p-persistent CSMA system can be obtained as*

$$\boldsymbol{\Lambda}_{\mathbf{p}} = \Big\{(\lambda_1,\lambda_2)|(\lambda_1,\lambda_2) \geq (0,0), (\lambda_1,\lambda_2) \textit{ lies below the curve } \lambda_2 = g(\lambda_1) \textit{ for } 0 \leq \lambda_1 < \mathcal{A}_{p_1}^{\max}\Big\}, \quad (32)$$

where

$$g(\lambda_1) = \begin{cases} \beta_{p_2}(\lambda_1), & \textit{for } 0 \leq \lambda_1 < \mathcal{A}_{\mathbf{p}}, \\ \tilde{\beta}_{p_1}(\lambda_1), & \textit{for } \mathcal{A}_{\mathbf{p}} \leq \lambda_1 < \mathcal{A}_{p_1}^{\max}, \end{cases} \quad (33)$$

with

$$\tilde{\beta}_{p_1}(\lambda_1) = \frac{q_{2|\{2\}}}{T} \cdot \frac{(1-\overline{\vartheta}_2 p_1)\left(1-(1-\overline{p}_1\overline{c})\frac{\lambda_1 T}{q_{1|\{1\}}p_1}\right)}{1-\vartheta_1(1-\overline{p}_1\overline{c})}. \quad (34)$$

Proof: See Appendix D. ■

We turn to find the (unconditional) stability region for $\mathbf{p} \in [0,1]^N$: The stability region $\boldsymbol{\Lambda}$ of the original system is obtained from the conditional stability region by

$$\boldsymbol{\Lambda} = \bigcup_{\mathbf{p}\in[0,1]^2} \boldsymbol{\Lambda}^*_{\mathbf{p}}. \quad (35)$$

Before proceeding further, let $\boldsymbol{\Lambda}^*_i$ denote the stability region of $\mathcal{S}^*_i$ for $i \in \{0,1,2\}$. The following theorem shows how $\boldsymbol{\Lambda}^*_i$ for $i \in \{0,1,2\}$ is related to each other.

Theorem 2. *The stability region of the original system with two users and that of the dominant systems are identical; that is, we have for all* $i \in \{0,1,2\}$,

$$\boldsymbol{\Lambda} = \boldsymbol{\Lambda}^* = \boldsymbol{\Lambda}^*_i = \bigcup_{\mathbf{p}\in[0,1]^2} \boldsymbol{\Lambda}^*_{\mathbf{p},i} = \bigcup_{\mathbf{p}\in[0,1]^2} \big\{(\lambda_1,\lambda_2)|\lambda_1 < \mathcal{A}_{\mathbf{p}}, \lambda_2 < \beta_{p_2}(\lambda_1)\big\}. \quad (36)$$

Proof: See Appendix E. ■

In contrast with $\boldsymbol{\Lambda}^*_{\mathbf{p},i}$ for $i \in \{0,1,2\}$, which has a distinct region, $\boldsymbol{\Lambda}^*_i$ is identical to each other, since $\boldsymbol{\Lambda}^*_i$ can be achieved by using vector $\mathbf{p} \in [0,1]$. While $\boldsymbol{\Lambda}$ is the union of all $\boldsymbol{\Lambda}^*_i$'s, $\boldsymbol{\Lambda}$ is also identical to any of $\boldsymbol{\Lambda}^*_i$. We shall depict $\boldsymbol{\Lambda}$ for some specific two-user CSMA systems in Section IV. It is also notable that the stability region for two classes of users is obtained by the MFA in [21], [22], and [24], which assumed that the users' queueing process becomes decoupled when they are saturated. Theorem 2 validates that their assumption is true for the stability region of two-user system.

The following theorem finds the stability region explicitly.

Theorem 3. *The stability region of a two-user MPR CSMA system is described as*

$$\boldsymbol{\Lambda} = \{(\lambda_1,\lambda_2)|(\lambda_1,\lambda_2) \geq (0,0), (\lambda_1,\lambda_2) \textit{ lies below the curve } \lambda_2 = h(\lambda_1)\}, \quad (37)$$

in which for $\vartheta_1 + \vartheta_2 \geq 1$ *(strong-reception capability) we have*

$$h(\lambda_1) = \begin{cases} \frac{q_{2|\{2\}}}{T}\left(1-\frac{\overline{\vartheta}_2}{\vartheta_1}\cdot\frac{\lambda_1 T}{q_{1|\{1\}}}\right), & \textit{for } 0 \leq \lambda_1 \leq \frac{q_{1|\{1,2\}}}{T}, \\ \frac{q_{2|\{2\}}}{T}\frac{\vartheta_2}{\overline{\vartheta}_1}\left(1-\frac{\lambda_1 T}{q_{1|\{1\}}}\right), & \textit{for } \frac{q_{1|\{1,2\}}}{T} < \lambda_1 < \frac{q_{1|\{1\}}}{T}. \end{cases} \quad (38)$$

In particular, for $\lambda_1 = \frac{q_{1|\{1,2\}}}{T}$ *in (38), we have* $h(\lambda_1) = \frac{q_{2|\{2\}}}{T}\vartheta_2 = \frac{q_{2|\{1,2\}}}{T}$.

For $\vartheta_1 + \vartheta_2 < 1$ (weak-reception capability), the envelope is found as

$$h(\lambda_1) = \begin{cases} \frac{q_{2|\{2\}}}{T}\left(1 - \frac{\overline{\vartheta}_2}{\vartheta_1} \cdot \frac{\lambda_1 T}{q_{1|\{1\}}}\right), & \text{for } 0 \le \lambda_1 \le u_1, \\ J(\lambda_1), & \text{for } u_1 < \lambda_1 < u_2, \\ \frac{q_{2|\{2\}}}{T}\frac{\vartheta_2}{\overline{\vartheta}_1}\left(1 - \frac{\lambda_1 T}{q_{1|\{1\}}}\right), & \text{for } u_2 \le \lambda_1 < \frac{q_{1|\{1\}}}{T}, \end{cases} \tag{39}$$

in which u_1 and u_2 are given in (82) and (86) in Appendix F. respectively. In (39), $J(\lambda_1)$ is expressed as

$$J(\lambda_1) = \frac{q_{1|\{1\}}}{T} \cdot \frac{1}{(1-\vartheta_1 c)^2}\left\{\sqrt{c}\sqrt{\frac{\lambda_1 T}{q_{1|\{1\}}}\left(\vartheta_1 + \vartheta_2 + \vartheta_1\vartheta_2 c\right)} - \sqrt{c\overline{\vartheta}_1 + \overline{c}\left(1 - \frac{\lambda_1 T}{q_{1|\{1\}}}\right)}\right\}^2.$$

Proof: See Appendix F. ■

Notice that the stability region of the strong and weak MPR channels includes linear boundaries in λ_1 and λ_2. However, they have different slopes due to $\vartheta_1 + \vartheta_2 > 1$ for the strong MPR and $\vartheta_1 + \vartheta_2 < 1$ for the weak MPR. Fig. 4 sketches the unconditional stability region of CSMA system with the strong, weak, and no MPR channels according to Theorem 3. In particular, the stability region of CSMA system without MPR channel is found in [6]; that is, $(\lambda_1 + \lambda_2)T + 2\sqrt{\sigma T \lambda_1 \lambda_2} = 1$ for $q_{1|\{1\}} = q_{2|\{2\}} = 1$. It shows how much an MPR channel can enlarge the stability region of CSMA systems. This will be discussed again in Section IV.

As a remark, from the results in [24] and [27] and Theorems 2 and 3, we can extend the stability region of the two-user system into the N-user system with multiple classes without rigorous proof for further discussion; its validity should be further investigated as future work.

Remark 1. *Suppose J classes (or group) of users, each of which has N_j users with identical arrival rate λ_j and (re)transmission probability p_j for $j \in \{1, 2, \ldots, J\}$. Let q_i be the probability that i packets are successfully received when i packets are transmitted at the same time, in which we have $q_i = 0$ for $i > M$. Thus, $n_i q_{n_1+n_2+\cdots+n_J}$ indicates that n_i packets from class i are successfully transmitted when the sum of the packets from all classes, i.e., $n_i + n_2 + \cdots + n_J \le M$, are transmitted. The boundary of the stability region for this system, denoted by $\overline{\mathcal{B}}$, is the boundary of the following set $\overline{\mathcal{S}}$:*

$$\overline{\mathcal{S}} = \left\{(N_j\lambda_j)|N_j\lambda_j = \frac{S_j}{\sigma P_I + T(1-P_I)}\right\}, \tag{40}$$

where P_I is the probability of the idle channel:

$$P_I = \prod_{j=1}^{J}(1-p_j)^{N_j}. \tag{41}$$

Note that S_j denotes the average number of packets from class j that are successfully transmitted during the average renewal period:

$$S_j = \sum_{\substack{n_1+\cdots+n_J \le M \\ 0 \le n_j \le M}} n_j q_{n_1+\cdots+n_J} \prod_{i=1}^{J}\binom{N_i}{n_i} p_i^{n_i}(1-p_i)^{N_i-n_i}. \tag{42}$$

For $M = 2$, we can relate $q_1 = q_{1|\{1\}} = q_{2|\{2\}}$ and $q_2 = q_{1|\{1,2\}} = q_{2|\{1,2\}}$. This remark will be discussed in Section IV.

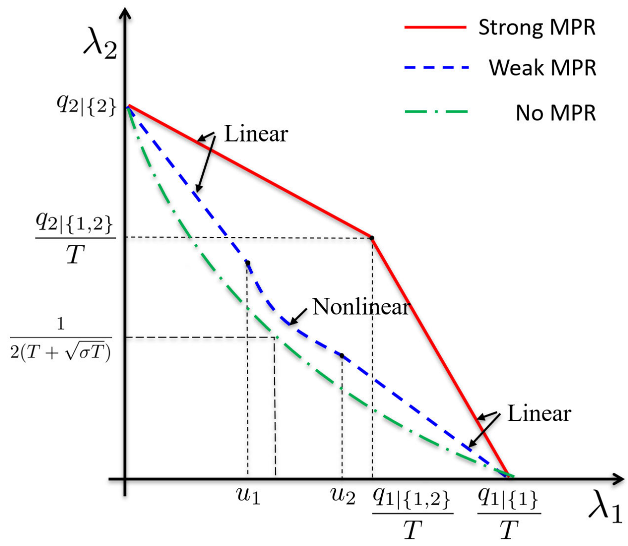


FIGURE 4. Unconditional stability region of a two-user *p*-persistent CSMA system.

A. STABILITY REGION FOR N SYMMETRIC USERS, N > 2

In this section, let us consider the MPR CSMA system, where N users have the same packet arrival rate and employ an identical retransmission probability p. As in [15] and [32], we assume a specific but widely adopted MPR channel that as long as the number of transmitting users is less than or equal to M, the users make a successful transmission.

Theorem 4. *For a symmetric N-user CSMA system, i.e., $\lambda = \lambda_i$, and $p = p_i$ for all i, the system is stable, if we have*

$$\lambda < \max_{0 \le p \le 1} \mathcal{H}_M(p, N)/N, \tag{43}$$

where $\mathcal{H}(p, N)$ is expressed as

$$\mathcal{H}_M(p,N) = \frac{\sum_{i=1}^{M} i\mathfrak{B}_i^N(p)}{\sigma\mathfrak{B}_0^N(p) + T\sum_{i=1}^{M}\mathfrak{B}_i^N(p) + T\left(1 - \sum_{k=0}^{M}\mathfrak{B}_k^N(p)\right)}, \tag{44}$$

in which $\mathfrak{B}_k^N(p)$ denotes a binomial distribution with parameters, N and p, i.e., $\mathfrak{B}_k^N(p) = \binom{N}{k}p^k(1-p)^{N-k}$ for $0 \le k \le N$. Otherwise, the system is unstable.

Algorithm 1 Pseudo-Bayesian Backoff Algorithm

1. Initialize $\lambda_0 = 0$, $\nu_0 = 1, \theta = 0.99$, and do the following at t_k.
2. **if** the current slot is idle or success **then**
3. $\quad \nu_{t_k} = \max\left(\nu_{t_{k-1}} - x_M^* + m - m_b, 0\right)$
4. **else if** Collision **then**
5. $\quad \nu_{t_k} = \nu_{t_{k-1}} + \Xi(x_M^*, M)$
6. **end if**
7. $\lambda_{t_k} = \theta\lambda_{t_{k-1}} + (1-\theta)\dfrac{m_b}{b_{t_k} - b_{t_{k-1}}}$
8. $\nu_{t_k} = \nu_{t_k} + \lambda_{t_k}$
9. $p_{t_k} = \min(x_M^*/\nu_{t_k}, 1)$.

TABLE 1. The values of x_M^*, $\Xi(x_M^*, M)$, and $\Theta(x)$ for different M's.

M	1	2	3	4	5
x_M^*	0.3046	0.9318	1.8166	2.6691	3.4753
$\mathcal{H}_M(p, 15)$	0.7451	1.1439	1.6068	2.1672	2.8072
$\mathbf{\Theta}_M(x_M^*)$	0.7375	1.1278	1.5580	2.0587	2.6136
$\Xi(x_M^*, M)$	1.8022	2.3363	2.6491	2.9521	3.2660

Proof: The numerator in (44) shows the average number of packets successfully transmitted, whereas the denominator is the average renewal cycle. Accordingly, $\mathcal{H}(p, N)$ represents the throughput of symmetric N-user system conditioned on p, when all the users have a packet to send, i.e., stochastic dominant system. The overall inputs to this system should be less than the throughput for the stochastic dominant system to be stable. Then, the conditional stability region of a symmetric CSMA system is expressed as

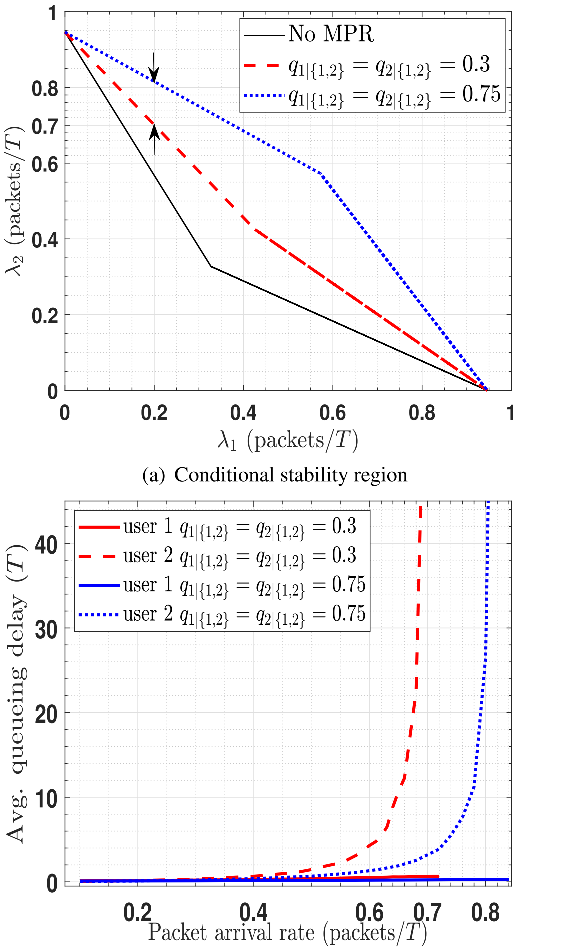
$$\mathbf{\Theta}_p = N\lambda < \mathcal{H}_M(p, N). \tag{45}$$

In this system, the stochastic dominant system becomes unstable if $N\lambda \geq \mathcal{H}(p, N)$. Finally, the original system would be indistinguishable from the stochastic dominant system as the arrival rate becomes closer to each user's throughput. The unconditional stability region is now expressed as (43). ∎

In order to investigate the asymptotic analysis for (43), let $x = pN$ denote the average transmission attempt. As N goes to infinity, this system is stable if

$$\begin{aligned}\mathbf{\Theta}_M(x) &< \max_x \lim_{N\to\infty} \mathcal{H}_M(x/N, N)\\ &\approx \max_x \frac{\sum_{i=1}^M i\phi_i(x)}{\sigma e^{-x} + T\sum_{i=1}^M \phi_i(x) + T\left(1 - \sum_{i=0}^M \phi_i(x)\right)}\\ &= \max_x \frac{\sum_{i=1}^M i\phi_i(x)}{(\sigma - T)e^{-x} + T}.\end{aligned} \tag{46}$$

We can maximize $\mathbf{\Theta}_M(x)$ with respect to x. Let x_M^* be the maximizer of (46). Table 1 presents x_M^* and $\mathbf{\Theta}_M(x_M^*)$. We compare the maximum of $\mathcal{H}_M(p, 15)$ for $N = 15$, i.e., throughput with 15 users with respect to p. It can be seen that $\mathbf{\Theta}_M(x_M^*)$ for a large population is slightly lower than $\mathcal{H}_M(p, 15)$ for a small population. Additionally, $\Xi(x_M^*, M)$ is used in Algorithm 1, which will be discussed in the next section.



(a) Conditional stability region

(b) Average queueing delay with $\lambda_1 = 0.2$.

FIGURE 5. Conditional stability region and its validation, $p_1 = p_2 = 0.5$.

B. BACKOFF ALGORITHM FOR N-USER SYSTEM

Suppose n_{t_k} backlogged users present in the system at embedded point t_k. We want the users to employ a backoff algorithm in order to control (re)transmission probability p of maximizing throughput $\mathcal{H}_M(p, n_{t_k})$. To do this, each user should know n_{t_k} to realize throughput-optimal p. However, it is very hard for them to know it. In the backoff algorithm presented in Algorithm 1, the AP estimates *its mean value*, i.e., $\mathbb{E}[n_{t_k}]$, which is denoted by ν_{t_k}, and then broadcasts throughput-optimal p_{t_k} at the embedded point t_k. Then, the backlogged users can use p_{t_k}. The derivation for each update equation on ν_{t_k} is detailed in Appendix G. Here, we discuss how it works.

Once they sense the channel idle, the users with a non-empty queue (re)transmit their packet with probability $p_{t_{k-1}}$, which has been broadcast at embedded time t_{k-1} by the AP. Notice that the AP can broadcast this only at embedded points since the users read the downlink broadcast message once sensing the channel. Users who transmit the last packet in their queue add one bit to it, say $b_e \in \{0, 1\}$, to inform the AP that their queue becomes empty. If they make a successful transmission, the AP adds up b_es. The sum of

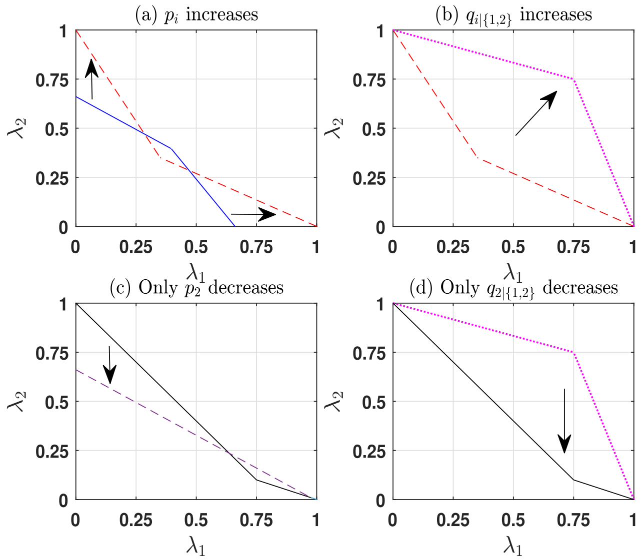


FIGURE 6. Conditional stability regions subject to user's parameters.

TABLE 2. MAC layer parameters.

σ	9 μsec	SIFS	16 μsec
DIFS	34 μsec	ACKTime	44 μsec
Packet TX : 100 bytes	64 μsec	ACK Time-out	94 μsec

these bits is denoted by m_b. This shows how many users' queue becomes empty, which should be subtracted from the backlog estimation as in line 3. Notice that if the users make a successful transmission, but their queue is not empty, they are still backlogged users.

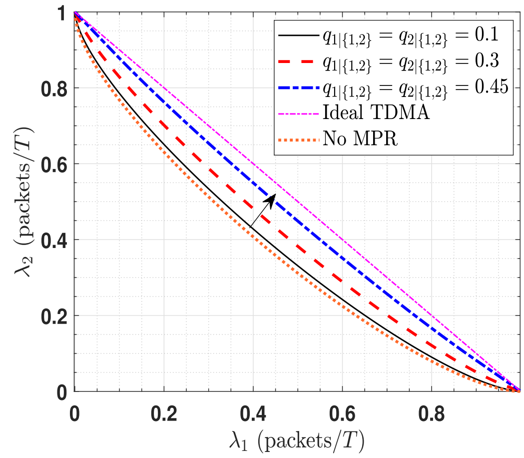
Depending on the channel outcome, the AP updates ν_{t_k} as shown in lines 3 and 5. In line 5 of Algorithm 1 and in Table 1, $\Xi(x^*_M)$ is a correction factor for ν_{t_k} upon collision.

Let b_{t_k} be the time epoch when at least one user with the last packet in its queue successfully transmits it. Thus, the interval $b_{t_k} - b_{t_{k-1}}$ indicates the time period that $m_b > 0$ occurs. In line 7, the system estimates the mean rate at which a user with an empty queue has a packet to send, which increases the backlog size. The parameter θ is a weighting factor that balances the previously estimated and newly observed rates based on the recent backlog size. Here, it is notable that the leaving rate out of the system would be equal to the joining rate to the system in steady-state.

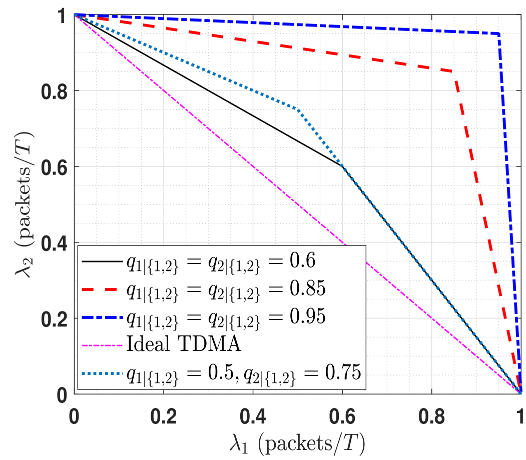
IV. NUMERICAL RESULTS

In our numerical studies, to set T and σ, we use (7) and the values in Table 2, where MAC layer parameters for IEEE 802.11a/g are listed when the transmission data rate at the physical layer is set to 24 Mbps. When normalizing the system parameter by T, we have $\sigma = 0.057$. Regarding simulation for queueing performance, each simulation run time is 10^7 (sec), and the mean of five time-averaged results are presented.

Understanding conditional stability region: In Figs. 5-6, we examine and simulate the conditional stability region as $q_{i|\{1,2\}}$ increases.



(a) Stability region of weak MPR with $\sigma = 0.057$.

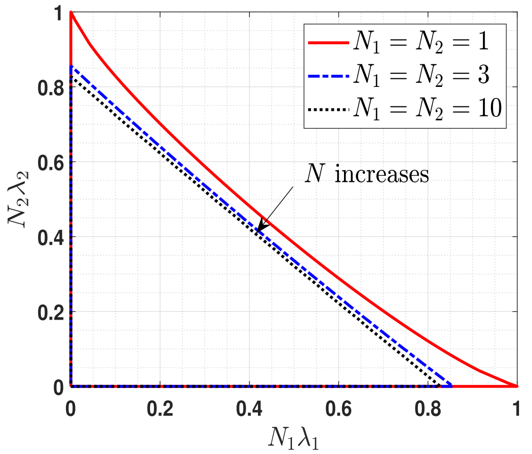


(b) Stability region of strong MPR with $\sigma = 0.057$.

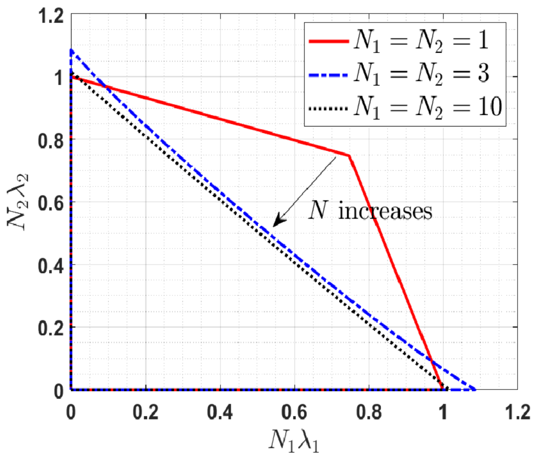
FIGURE 7. Stability region of MPR CSMA system.

First, consider $\lambda_1 = 0.2$ in Fig. 5(a), which is derived from (32). As long as λ_2 is less than 0.7 for $q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.3$, the conditional stability region means that both users' queue length will remain finite. If $q_{1|\{1,2\}}$ and $q_{2|\{1,2\}}$ are changed from 0.3 to 0.75, their queue remains finite as λ_2 increases up to 0.815. This behavior is further illustrated in Fig. 5(b), where user 2's queue length grows explosively as λ approaches 0.7 for $q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.3$ or λ approaches 0.8 for $q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.75$. Notably, this behavior in simulation aligns with our analytical results from the conditional stability regions in Fig. 5(a).

Let us recall in Figs. 5(a) and 6 that $\lambda_2 = \mathcal{B}^{\max}_{p_2}$ for $\lambda_1 = 0$ and $\lambda_1 = \mathcal{A}^{\max}_{p_1}$ for $\lambda_2 = 0$. In Fig. 6(a), we set $q_{1|\{1,2\}} = q_{2|\{1,2\}} = 0.3$ and $p_1 = p_2 = 0.1$ and increase p_1 and p_2 up to 1. As either or both p_1 and p_2 become small, $\mathcal{A}^{\max}_{p_1}$ and $\mathcal{B}^{\max}_{p_2}$ decrease. This shrunk the conditional stability region. If $q_{1|\{1,2\}}$ and $q_{2|\{2\}}$ are increased to 0.75 in Fig. 6(b), the conditional stability region become much large. For $q_{2|\{1,2\}} = 0.1$ in Fig. 6(d), i.e., user 2's transmission becomes less successful upon both users' transmission, the conditional stability region of user 2 gets significantly



(a) Stability region of weak MPR with $\sigma = 0.057$ and $q_2 = 0.3$.



(b) Stability region of strong MPR with $\sigma = 0.057$ and $q_2 = 0.75$.

FIGURE 8. **Stability region of MPR CSMA system for two groups of users.**

TABLE 3. **Comparison of x^*_M, and $\Theta_M(x)$ in MPR S-ALOHA.**

M	1	2	3	4	5
x^*_M	1	1.618	2.2695	2.9451	3.6395
$\Theta^\dagger_M(x^*_M)$	0.3679	0.84	1.3711	1.9424	2.5435

smaller. In Fig. 6(c), if only p_2 decreases back to 0.5, user 2's region gets further lowered.

Characterization of stability region: Fig. 7 depicts the stability region as $q_{i|\{1,2\}}$ for $i \in \{1, 2\}$ increases. For comparison, we consider the stability region of an (ideal) time division multiple access (TDMA) system with optimal time-sharing of a slot as

$$\mathfrak{C}_{\text{TD}} = \bigcup_{0\le\hat{\theta}\le 1} \{(\lambda_1, \lambda_2)|\lambda_1 < \hat{\theta} q_{1|\{1\}}, \lambda_2 < q_{2|\{2\}}(1-\hat{\theta})\}, \tag{47}$$

where $\hat{\theta}$ denotes the time proportion of a slot assigned to user 1. As expected, when $q_{i|\{1,2\}}$ increases, the stability region becomes larger, particularly for the region for $\lambda_1 = \lambda_2$.

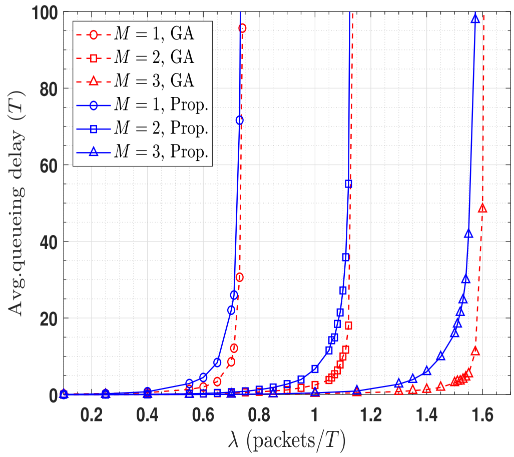
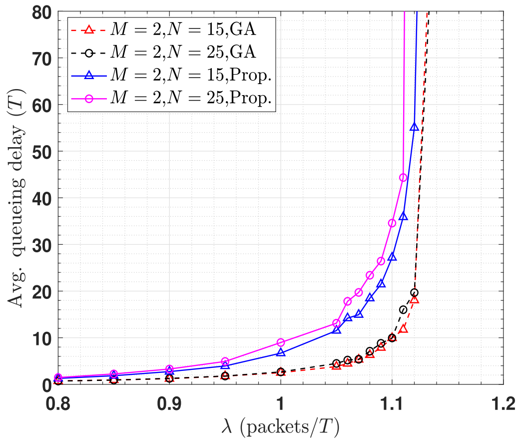


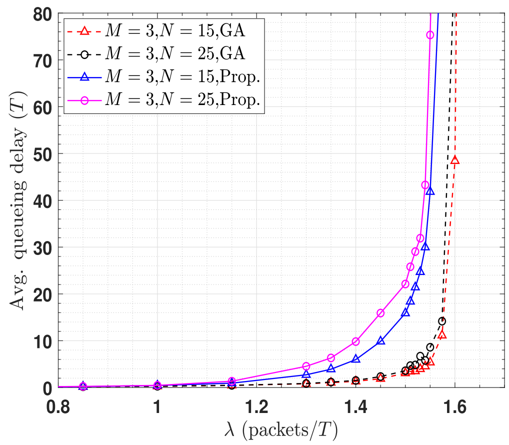
FIGURE 9. **Queueing delay with different M's ($N = 15$).**

Note that even for the systems without MPR, as $\sigma \to 0$, i.e., instantaneous channel sensing capability and zero redundancy of channel sensing, its stability region becomes equal to that of ideal TDMA [6]. When MPR channel becomes stronger in Fig. 7(b), the rectangular region for $0 \le \lambda_i \le 1$ for $i \in \{1, 2\}$ is the stability region. Especially, if $q_{2|\{1,2\}} > q_{1|\{1,2\}}$, the region for user 2 is slightly increased. From [4] and [6], the stability region of CSMA (without MPR) is much larger than that of S-ALOHA. However, Fig. 7(b) shows that when the strong MPR channel is considered in CSMA system, its stability region might not be much different from the strong MPR S-ALOHA. In other words, advanced signal processing capability in the physical layer might enhance a low throughput of S-ALOHA. For strong-MPR CSMA with a very small number of users, sensing might not significantly improve the system throughput.

In Fig. 8, we depict the boundary of the stability region of the systems with two groups of users as in [24] and [27], based on Remark 1. Each group has N_i users who utilize retransmission probability p_i for $i \in \{1, 2\}$. Furthermore, we assume $M = 2$, $q_1 = 1$, and $q_2 = 0.3$ as a weak MPR in Fig. 8(a), and $q_2 = 0.75$ as a strong MPR in Fig. 8(b), respectively. As N_i increases, generally, the stability region gets shrunk. For $N_i = 10$, the boundary of the stability region becomes a straight line both in weak and strong MPR channel. The boundary point of each axis indicates the maximum arrival rate that each group can accommodate under stability when the other group does not transmit at all. Moreover, for a strong MPR channel, the boundary of the stability region with $N_i = 3$ is slightly larger than that with $N_i = 1$ as any of $N_i\lambda_i \to 0$. The reason is that for the strong MPR channel with $q_2 = 0.75$, only two users are insufficient to fully exploit MPR channel capacity. More precisely, for a two-user system with $\lambda_i = 0$, only one user can make a successful transmission. However, as N_i increases slightly, if one group of users may not attempt to transmit at all, more than one user in another group can enjoy successful transmissions.



(a) Queueing delay with $M = 2$



(b) Queueing delay with $M = 3$

FIGURE 10. Queueing delay with different population sizes.

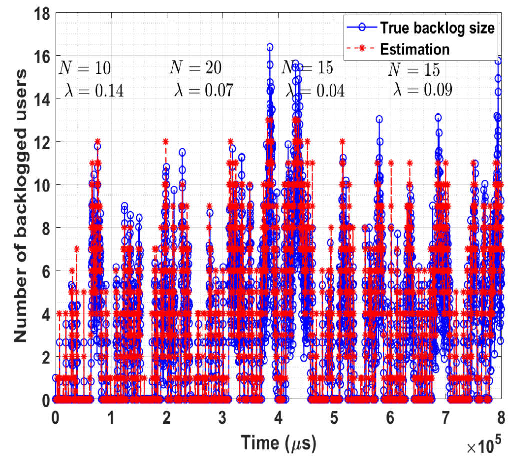


FIGURE 11. Estimation by the proposed backoff algorithm.

Interestingly, for the strong MPR channel, as N increases, the stability region of MPR CSMA becomes that of ideal TDMA in (47).

Effect of MPR capability M: In order to validate Theorem 4, i.e., the stability condition for N symmetric users, as MPR capability M increases, we observe the average queueing delay for $N = 15$, where the users employ Algorithm 1. As a benchmark system, we consider a genie-aided (GA) system [36], where the AP can get the exact backlog size n_{t_k}, instead of estimation on $\mathbb{E}[n_{t_k}]$. Thus, the GA system can yield the lowest queueing delay.

For each M, the maximum throughput or stability condition $\boldsymbol{\Theta}_M(x_M^*)$ is given in Table 1. As λ goes closer to $\boldsymbol{\Theta}_M(x_M^*)$ in Fig. 9, the queueing delay rises explosively. It can be seen that the proposed algorithm keeps the system stable reasonably well. Let us compare the throughput of MPR CSMA with MPR S-ALOHA. For M-MPR capability, the throughput of MPR S-ALOHA for a large population can be expressed as

$$\boldsymbol{\Theta}_M^{\dagger}(x) = \sum_{i=1}^{M} i\phi_i(x), \tag{48}$$

where $x = Np$. Table 3 shows the maximizer x_M^* for (48) and the maximum of (48) with x_M^*. For a large population, as M increases, the difference between the maximum throughput of MPR CSMA and that of MPR S-ALOHA seems to be vanishing.

Effect of population size: In Figs. 10(a)-10(b), we consider the average queueing delay by increasing the number of users for $M = 2$ and 3. Notice that for the GA system that has perfect knowledge of the backlog size, even though the population size changes, the performance remains almost the same since it makes use of the exact backlog information. However, as the population size increases from 15 to 25, the queueing delay slightly increases with the proposed algorithm.

Fig. 11 depicts the estimated backlog size by the proposed algorithm and the exact backlog size for $M = 3$. The system starts with $N = 10$ and $\lambda = 0.14$ (the packet arrival rate per user). At $t = 2000$, ten more users join, but λ is reduced to 0.07. Further, at $t = 4000$, five users leave the system, while λ decreases, as shown in Fig. 11. At $t = 6000$, λ increases. We can see that the proposed algorithm properly tracks the true backlog size, even as the population and packet arrival rates change over time.

V. CONCLUSION

This work characterized the stability regions of MPR p-persistent CSMA systems using the stochastic dominant system. The first result is that we showed how the stability could be enlarged or shrunk according to the users' parameters for the systems with two asymmetric users. If the MPR channel is weak, i.e., $q_{1,2|\{1,2\}} < q_{1,\underline{2}|\{1,2\}} \leq 1$, an optimized CSMA without an MPR channel might be as good as weak-MPR CSMA. Moreover, for strong-MPR CSMA, we came to the conclusion that channel sensing could be relaxed since S-ALOHA with the same MPR capability could show a performance close to it. A second result is that we obtained the stability condition of the system with N symmetric users and developed the queue-stabilizing backoff algorithm.

It was demonstrated that the proposed algorithm ensures the stability of the system.

APPENDIX A PROOF OF LEMMA 1

We can find (23) by the renewal reward theorem [35]; that is, L represents the expected cycle length of a renewal, while $p_{s,i}$ indicates the average reward obtained during L. From (21), we have

$$\lambda_1 < \frac{p_{s,1}}{L} = \frac{q_{1|\{1\}}p_1\overline{p}_2 + q_{1|\{1,2\}}p_1p_2}{T\left(c\overline{p}_1\overline{p}_2 + 1 - \overline{p}_1\overline{p}_2\right)} = \frac{q_{1|\{1\}}}{T}\frac{p_1(1-\overline{\vartheta}_1 p_2)}{1-\overline{c}\cdot\overline{p}_1\overline{p}_2} \equiv \mathcal{A}_{\mathbf{p}}. \tag{49}$$

This means that the packet arrival rate for user 1 should not be greater than throughput per renewal cycle. Similarly, for user 2, we can find

$$\lambda_2 < \frac{p_{s,2}}{L} = \frac{q_{2|\{2\}}p_2\overline{p}_1 + q_{2|\{1,2\}}p_1p_2}{T\left(\frac{\sigma}{T}\overline{p}_1\overline{p}_2 + 1 - \overline{p}_1\overline{p}_2\right)} = \frac{q_{2|\{2\}}}{T}\frac{p_2(1-\overline{\vartheta}_2 p_1)}{1-\overline{c}\cdot\overline{p}_1\overline{p}_2} \equiv \mathcal{B}_{\mathbf{p}}. \tag{50}$$

APPENDIX B PROOF OF LEMMA 2

In system $\mathcal{S}_2^*$, where user 2 transmits dummy packet, user 1's queue can be stabilized if $\lambda_1 < \mathcal{A}_{\mathbf{p}}$. Since user 1 is not a dominant user in $\mathcal{S}_2^*$, user 2's packet transmission is interfered by user 1 only when $Q_1 > 0$. To observe this event explicitly, let us denote by π_0 the probability that user 1's queue is empty at an embedded point, which is given by (56) in Appendix C.

Suppose that user 1's queue is stable, which represents $\pi_0 > 0$. Then, the packet transmission success of user 2 occurs with probability $q_{2|\{2\}}p_2\pi_0 + p_{s,2}\overline{\pi}_0$, which shows the dependence on the state of user 1's queue. By applying Loynes' theorem, we have

$$\lambda_2 < \frac{q_{2|\{2\}}p_2\pi_0 + p_{s,2}\overline{\pi}_0}{\pi_0(\overline{p}_2\sigma + p_2T) + \overline{\pi}_0 L} = \beta_{p_2}(\lambda_1), \tag{51}$$

where the numerator represents the probability that a packet transmission of user 2 is successful and the denominator shows the average time between two consecutive embedded points seen by user 2.

When substituting π_0 in (56) into (51) and rearranging (51) with respect to λ_1, we can write the RHS of (51) as a function of λ_1 by (52), as shown at the bottom of the next page.

Similarly, for stochastic dominant system $\mathcal{S}_1^*$, where user 1 is a dominant user, we can obtain $\mathbf{\Lambda}_{\mathbf{p},1}^*$ by

$$\mathbf{\Lambda}_{\mathbf{p},1}^* = \{(\lambda_1,\lambda_2)|\lambda_1 < \alpha_{p_1}(\lambda_2), \lambda_2 < \mathcal{B}_{\mathbf{p}}\},$$

in which $\alpha_{p_1}(\lambda_2)$ is expressed as

$$\alpha_{p_1}(\lambda_2) = \frac{q_{1|\{1\}}p_1\phi_0 + p_{s,2}\overline{\phi}_0}{\phi_0(\overline{p}_1\sigma + p_1T) + \overline{\phi}_0 L}. \tag{53}$$

Note that ϕ_0 is the probability that user 2's queue length is zero at an embedded point. This ϕ_0 is also given by (57) in Appendix C. The same argument for conditional stability region $\mathcal{S}_2^*$ can be applied to $\mathcal{S}_1^*$.

APPENDIX C QUEUEING BEHAVIOR FOR USER 1 IN $\mathcal{S}_2^*$

We assume that k packets arrive at user 1's queue according to a Poisson distribution with the mean rate λ_1 (packets/μsec) during x μsec. Since it takes σ (μsec) for an idle channel and T (μsec) otherwise, user 1's queue is an M/G/1 queueing system. Using Pollaczek-Khinchine (PK) formula for M/G/1 queueing system, we can write the probability generating function (PGF) for the queue length distribution as

$$\Pi(z) = \frac{(1-z)B^*((1-z)\lambda)\pi_0}{B^*((1-z)\lambda) - z}, \tag{54}$$

where π_0 is the probability of the empty queue, and $B^*(s)$ denotes the Laplace transform of the service time distribution for user 2's queue. We can find it as

$$\begin{aligned} B^*(s) &= p_{s,1}e^{-sT}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\binom{i+k}{k}\left(\overline{p}_1\overline{p}_2e^{-s\sigma}\right)^i \\ &\quad\times\left(\left(1-\overline{p}_1\overline{p}_2-p_{s,1}\right)e^{-sT}\right)^k \\ &= p_{s,1}e^{-sT}\sum_{n=0}^{\infty}\binom{n}{k}\left(\overline{p}_1\overline{p}_2e^{-s\sigma}\right)^{n-k} \\ &\quad\times\left(\left(1-\overline{p}_1\overline{p}_2-p_{s,1}\right)e^{-sT}\right)^k \\ &= \frac{p_{s,1}e^{-sT}}{1-\left[\overline{p}_1\overline{p}_2e^{-s\sigma} + \left(1-\overline{p}_1\overline{p}_2-p_{s,1}\right)e^{-sT}\right]}. \end{aligned} \tag{55}$$

From $\lim_{z\to 1}\Pi(z) = 1$, we have π_0 as

$$\pi_0 = \frac{p_{s,1} - L\lambda_1}{p_{s,1} - L\lambda_1 + (\overline{p}_2\sigma + p_2T)\lambda_1}. \tag{56}$$

Since a stable system has $\pi_0 > 0$, we have $\lambda_1 < p_{s,1}/L$ in (56); this agrees with Loynes' theorem. Similarly using PK formula, we can also obtain the steady-state probability that user 2's queue is empty in $\mathcal{S}_1$ as

$$\phi_0 = \frac{p_{s,2} - L\lambda_2}{p_{s,2} - L\lambda_2 + (\overline{p}_1\sigma + p_1T)\lambda_2}. \tag{57}$$

APPENDIX D PROOF OF THEOREM 1

We can rewrite $\mathbf{\Lambda}_{\mathbf{p}}$ as

$$\begin{aligned} \mathbf{\Lambda}_{\mathbf{p}} &= \mathbf{\Lambda}_{\mathbf{p},1}^* \bigcup \mathbf{\Lambda}_{\mathbf{p},2}^* \\ &= \{(\lambda_1,\lambda_2)|\lambda_1 < \mathcal{A}_{\mathbf{p}}, \lambda_2 < \beta_{p_2}(\lambda_1)\} \\ &\quad \bigcup\{(\lambda_1,\lambda_2)|\lambda_1 < \alpha_{p_1}(\lambda_2), \lambda_2 < \mathcal{B}_{\mathbf{p}}\}, \end{aligned} \tag{58}$$

where (26) and (27) are used for $\mathbf{\Lambda}_{\mathbf{p},1}^*$ and $\mathbf{\Lambda}_{\mathbf{p},2}^*$, respectively.

Since we have $\mathcal{A}_{\mathbf{p}} \le \alpha_{p_1}(\lambda_2)$ and $\mathcal{B}_{\mathbf{p}} \le \beta_{p_2}(\lambda_1)$, λ_2 is upper-bounded by $\beta_{p_2}(\lambda_1)$ for $\lambda_1 < \mathcal{A}_{\mathbf{p}}$; on the other hand, for $\mathcal{A}_{\mathbf{p}} \le \lambda_1 < \alpha_{p_1}(\lambda_2)$, λ_2 is upper-bounded by $\mathcal{B}_{\mathbf{p}}$. Note that the maximum of $\alpha_{p_1}(\lambda_2)$ can be $\mathcal{A}_{p_1}^{\max}$. From $\lambda_1 < \alpha_{p_1}(\lambda_2)$ in (53), i.e.,

$$\begin{aligned} \lambda_1 &< \alpha_{p_1}(\lambda_2) \\ &= \frac{q_{1|\{1\}}p_1}{T}\left(\frac{1}{1-\overline{p}_1\overline{c}} + \left(\vartheta_1 - \frac{1}{1-\overline{p}_1\overline{c}}\right)\frac{\frac{\lambda_2 T}{q_{2|\{2\}}}}{1-\overline{\vartheta}_2p_1}\right). \end{aligned} \tag{59}$$

By solving (90) with respect to λ_2, we can obtain another upper-bound for λ_2 as

$$\lambda_2 < \tilde{\beta}_{p_1}(\lambda_1) \equiv \frac{q_{2|\{2\}}}{T} \cdot \frac{(1-\overline{\vartheta}_2 p_1)\left(1-(1-\overline{p}_1\overline{c})\frac{\lambda_1 T}{q_{1|\{1\}}p_1}\right)}{1-\vartheta_1(1-\overline{p}_1\overline{c})}. \tag{60}$$

Therefore, we have

$$\lambda_2 \le \min\left(\tilde{\beta}_{p_1}(\lambda_1), \mathcal{B}_{\mathbf{p}}\right) = \tilde{\beta}_{p_1}(\lambda_1). \tag{61}$$

Summarizing the above, we have $\mathbf{\Lambda_p}$ that is the region below the following curve:

$$g(\lambda_1) = \begin{cases} \beta_{p_2}(\lambda_1), & \text{if } 0 \le \lambda_1 < \mathcal{A}_{\mathbf{p}}, \\ \tilde{\beta}_{p_1}(\lambda_1), & \text{if } \mathcal{A}_{\mathbf{p}} \le \lambda_1 < \mathcal{A}_{p_1}^{\max}. \end{cases} \tag{62}$$

APPENDIX E PROOF OF THEOREM 2

We can obtain $\mathbf{\Lambda}_2^*$ and $\mathbf{\Lambda}_0^*$, respectively, as

$$\mathbf{\Lambda}_2^* = \bigcup_{\mathbf{p}\in[0,1]^2} \left\{(\lambda_1,\lambda_2)|\lambda_1 < \mathcal{A}_{\mathbf{p}}, \lambda_2 < \beta_{p_2}(\lambda_1)\right\} \tag{63}$$

and

$$\mathbf{\Lambda}_0^* = \bigcup_{\mathbf{p}\in[0,1]^2} \left\{(\lambda_1,\lambda_2)|\lambda_1 < \mathcal{A}_{\mathbf{p}}, \lambda_2 < \mathcal{B}_{\mathbf{p}}\right\}. \tag{64}$$

From $\lambda_1 < \mathcal{A}_{\mathbf{p}}$ in the above, by rewriting (25) with respect to p_1, we obtain

$$\frac{\lambda_1 T(1-\overline{p}_2\overline{c})}{q_{1|\{1\}}(1-\overline{\vartheta}_1 p_2)-\lambda_1 T\overline{p}_2\overline{c}} < p_1 \le 1. \tag{65}$$

Consequently, we have

$$\frac{\lambda_1 T(1-\overline{p}_2\overline{c})}{q_{1|\{1\}}(1-\overline{\vartheta}_1 p_2)-\lambda_1 T\overline{p}_2\overline{c}} < 1. \tag{66}$$

By solving the above inequality with respect to λ_1, we have

$$\lambda_1 < \frac{q_{1|\{1\}}(1-\overline{\vartheta}_1 p_2)}{T}. \tag{67}$$

For $\mathbf{\Lambda}_0^*$, since $\mathcal{B}_{\mathbf{p}}$ is a decreasing function of p_1 for $0 \le p_1$ and $p_2 \le 1$, the maximum value of λ_2, i.e., $\mathcal{B}_{\mathbf{p}}$, is achieved at the minimum value of p_1 which is constrained by (65). Thus, substituting the LHS of (65) into $\mathcal{B}_{\mathbf{p}}$, we obtain

$$\lambda_2 < \beta_{p_2}(\lambda_1), \tag{68}$$

which indicates that $\mathbf{\Lambda}_0^*$ can be expressed as (63). This proves $\mathbf{\Lambda}_0^* = \mathbf{\Lambda}_2^*$. Similarly, we can also prove $\mathbf{\Lambda}_1^* = \mathbf{\Lambda}_0^*$. Therefore, we have

$$\mathbf{\Lambda} = \mathbf{\Lambda}_0^* = \mathbf{\Lambda}_1^* = \mathbf{\Lambda}_2^* = \mathbf{\Lambda}^*. \tag{69}$$

APPENDIX F PROOF OF THEOREM 3

From (36), for a given λ_1, it can be seen that λ_2 is upper-bounded by $\beta_{p_2}(\lambda_1)$. In order to maximize $\lambda_2 = \beta_{p_2}(\lambda_1)$, we can obtain the following equivalent maximization problem:

$$\begin{aligned} &\underset{p_2}{\text{maximize}} && \beta_{p_2}(\lambda_1) \\ &\text{subject to} && 0 \le p_2 \le \min(1, p_u), \end{aligned} \tag{70}$$

where the constraint is from (67):

$$p_u = \frac{q_{1|\{1\}} - \lambda_1 T}{q_{1|\{1\}}\overline{\vartheta}_1}. \tag{71}$$

To find the solution of (70), we first need to find $\frac{d\beta_{p_2}(\lambda_1)}{dp_2} = 0$. The expression of $\frac{d\beta_{p_2}(\lambda_1)}{dp_2}$ is given by (72), as shown at the bottom of the next page.

Here, let us write the numerator of (72), i.e., $f(p_2)$:

$$\begin{aligned} f(p_2) = c(1-\overline{\vartheta}_1 p_2)^2 \\ + \frac{\lambda_1 T}{q_{1|\{1\}}}\left(\vartheta_2(c+\overline{c}p_2)^2 - (c+\overline{c}\,\overline{\vartheta}_1 p_2^2)\right). \end{aligned} \tag{73}$$

If $f(p_2) \ge 0$, $\beta_{p_2}(\lambda_1)$ is an increasing function of p_2 and, otherwise, $\beta_{p_2}(\lambda_1)$ is a decreasing function of p_2.

We consider the following two conditions.

A. STRONG MPR CHANNEL: $\vartheta_1 + \vartheta_2 \ge 1$

In (72), $f(p_2)$ is rewritten as

$$\begin{aligned} f(p_2) =& c\Bigg[(1-\overline{\vartheta}_1 p_2)^2 + \frac{\lambda_1 T}{q_{1|\{1\}}}\Big\{(\vartheta_2 c - 1) \\ &+ p_2\frac{\overline{c}}{c}\left(2\vartheta_2 c + (\overline{c}\vartheta_2 - \overline{\vartheta}_1)p_2\right)\Big\}\Bigg]. \end{aligned} \tag{74}$$

$$\begin{aligned} \beta_{p_2}(\lambda_1) &= \frac{q_{2|\{2\}}p_2 p_{s,1} - q_{2|\{2\}}p_1 p_2 T\lambda_1 + q_{2|\{1,2\}}p_1 p_2(\overline{p}_2\sigma + p_2 T)\lambda_1}{(\overline{p}_2\sigma + p_2 T)p_{s,1}} \\ &= \frac{q_{2|\{2\}}p_2(q_{1|\{1\}}\overline{p}_2 + q_{1|\{1,2\}}p_2) - q_{2|\{2\}}p_2 T\lambda_1 + q_{2|\{1,2\}}p_2(\overline{p}_2\sigma + p_2 T)\lambda_1}{(\overline{p}_2\sigma + p_2 T)(q_{1|\{1\}}\overline{p}_2 + q_{1|\{1,2\}}p_2)} \\ &= \frac{q_{2|\{2\}}p_2}{T}\left(\frac{1}{1-\overline{p}_2\overline{c}} + \frac{\vartheta_2\frac{\lambda_1 T}{q_{1|\{1\}}}}{1-\overline{\vartheta}_1 p_2} - \frac{\frac{\lambda_1 T}{q_{1|\{1\}}}}{(1-\overline{p}_2\overline{c})(1-\overline{\vartheta}_1 p_2)}\right). \end{aligned} \tag{52}$$

For $\frac{\lambda_1 T}{q_{1|\{1\}}} < 1$, the following inequality holds:

$$\begin{aligned} f(p_2) &\geq \frac{\lambda_1 Tc}{q_{1|\{1\}}}\Big\{(1-\overline{\vartheta}_1 p_2)^2 + (\vartheta_2 c - 1) \\ &\quad + p_2\frac{\overline{c}}{c}\big(2\vartheta_2 c + (\overline{c}\vartheta_2 - \overline{\vartheta}_1)p_2\big)\Big\} \\ &= \frac{\lambda_1 Tc}{q_{1|\{1\}}}\left\{(\vartheta_2 c - \overline{\vartheta}_1 p_2)\left(\frac{c+\overline{c}p_2}{c}\right) + p_2\vartheta_2\frac{\overline{c}}{c}(c+\overline{c}p_2)\right\} \\ &= \frac{\lambda_1 Tc}{q_{1|\{1\}}}\left(\frac{c+\overline{c}p_2}{c}\right)\big(\vartheta_2 c\overline{p}_2 + (\vartheta_1+\vartheta_2-1)p_2\big). \end{aligned} \tag{75}$$

It can be observed that if $\vartheta_1 + \vartheta_2 \geq 1$ (strong MPR), $f(p_2)$ is always greater than zero, so is $\frac{d\beta_{p_2}(\lambda_1)}{dp_2}$; that is, $\beta_{p_2}(\lambda_1)$ is an increasing function of p_2 for $0 \leq p_2 \leq 1$. Therefore, the maximum value of $\beta_{p_2}(\lambda_1)$ can be found for $p_2 = 1$. When plugging $p_2 = 1$ into (29), we have the first equation of (38). This is valid if we can see if $p_u \geq 1$ from (71):

$$p_u = \frac{q_{1|\{1\}} - \lambda_1 T}{q_{1|\{1\}}\overline{\vartheta}_1} \geq 1 \Rightarrow \lambda_1 \leq \frac{q_{1|\{1,2\}}}{T}. \tag{76}$$

On the other hand, if $p_u < 1$, then the maximum of $\beta_{p_2}(\lambda_1)$ occurs at $p_2 = p_u$. Substituting $p_2 = p_u$ into (29), we have the second equation of (38). Note that λ_1 can not be larger than $\frac{q_{1|\{1\}}}{T}$ due to $p_u > 0$. Therefore, we have (38).

B. WEAK MPR CHANNEL: $\vartheta_1 + \vartheta_2 < 1$

First, we prove that $f(p_2)$ is a decreasing function of p_2 in a range of $0 \leq p_2 \leq 1$:

$$\begin{aligned} \frac{df(p_2)}{dp_2} &= 2\left(c\overline{\vartheta}_1^2 + \vartheta_2\frac{\lambda_1 T}{q_{1|\{1\}}}\overline{c}^2 - \frac{\lambda_1 T}{q_{1|\{1\}}}\overline{c}\,\overline{\vartheta}_1\right)p_2 \\ &\quad + 2c\left(-\overline{\vartheta}_1 + \vartheta_2\frac{\lambda_1 T}{q_{1|\{1\}}}\overline{c}\right) \\ &= 2\Big(-c\overline{\vartheta}_1(1-\overline{\vartheta}_1 p_2) + \vartheta_2\frac{\lambda_1 T}{q_{1|\{1\}}}\overline{c}(\overline{c}p_2 + c) \\ &\quad - \overline{c}\frac{\lambda_1 T}{q_{1|\{1\}}}\overline{\vartheta}_1 p_2\Big) \\ &\leq 2c\overline{\vartheta}_1\left(-(1-\overline{\vartheta}_1 p_2) + \frac{\lambda_1 T}{q_{1|\{1\}}}\overline{c}\,\overline{p}_2\right) \\ &\leq -2c\overline{\vartheta}_1(1-\overline{\vartheta}_1 p_2)(1-\overline{c}\,\overline{p}_2) \\ &\leq 0. \end{aligned} \tag{77}$$

In (77), the first *inequality* results from the fact that $\vartheta_2 < 1 - \vartheta_1$ and the second *inequality* is obtained from $p_2 < p_u$.

It is easy to check that $f(p_2)$ is larger than zero for $p_2 = 0$:

$$f(p_2)\big|_{p_2=0} = c\left(1 - \frac{\lambda_1 T}{q_{1|\{1\}}}(1-\vartheta_2 c)\right) \geq 0. \tag{78}$$

On the other hand, for $p_2 = 1$, we have

$$f(p_2)\big|_{p_2=1} = c\vartheta_1^2 - \frac{\lambda_1 T}{q_{1|\{1\}}}(1-\vartheta_1-\vartheta_2+\vartheta_1 c). \tag{79}$$

Notice that $f(p_2)|_{p_2=1}$ in (79) can be larger or smaller than zero. Thus, we consider two cases:

1) $\frac{\lambda_1 T}{q_{1|\{1\}}} \leq \frac{c\vartheta_1^2}{1-(\vartheta_1+\vartheta_2)+\vartheta_1 c}$: In this case, we have $f(p_2)|_{p_2=1} \geq 0$. While $f(p_2)$ is a decreasing function of p_2 for $0 \leq p_2 \leq 1$ as proved by (77), we always have $f(p_2) \geq 0$ in this case. From (72) it shows that $\beta_{p_2}(\lambda_1)$ is an increasing function of p_2. Hence, the maximum value occurs at $p_2 = \min(1, p_u)$. Notice that

$$\begin{aligned} 1 - p_u = \frac{\frac{\lambda_1 T}{q_{1|\{1\}}} - \vartheta_1}{1-\vartheta_1} &\leq \frac{\frac{c\vartheta_1^2}{1-(\vartheta_1+\vartheta_2)+\vartheta_1 c} - \vartheta_1}{1-\vartheta_1} \\ &= \frac{\frac{-\vartheta_1[1-(\vartheta_1+\vartheta_2)]}{1-(\vartheta_1+\vartheta_2)+\vartheta_1 c}}{1-\vartheta_1} \leq 0, \end{aligned} \tag{80}$$

where in the first inequality we have used our assumption $\frac{\lambda_1 T}{q_{1|\{1\}}} \leq \frac{c\vartheta_1^2}{1-(\vartheta_1+\vartheta_2)+\vartheta_1 c}$.
According to (80), the maximum value of $\beta_{p_2}(\lambda_1)$ can be found at $p_2 = \min(p_u, 1) = 1$. Consequently, we have

$$\beta_{p_2}(\lambda_1) \leq \frac{q_{2|\{2\}}}{T}\left(1 - \frac{\overline{\vartheta}_2}{\vartheta_1}\cdot\frac{\lambda_1 T}{q_{1|\{1\}}}\right), \tag{81}$$

which is the first equation of (39). This is valid for $\lambda_1 \leq u_1$, where u_1 comes from our assumption for $\frac{\lambda_1 T}{q_{1|\{1\}}} \leq \frac{c\vartheta_1^2}{1-(\vartheta_1+\vartheta_2)+\vartheta_1 c} = \frac{c\vartheta_1^2}{\vartheta_1+\vartheta_2+\vartheta_1 c}$:

$$u_1 = \frac{q_{1|\{1\}}}{T}\frac{c\vartheta_1^2}{\vartheta_1+\vartheta_2+\vartheta_1 c}. \tag{82}$$

2) $\frac{\lambda_1 T}{q_{1|\{1\}}} > \frac{c\vartheta_1^2}{1-(\vartheta_1+\vartheta_2)+\vartheta_1 c}$: Opposite to the previous case, this case corresponds to $f(p_2)|_{p_2=1} < 0$. Since $f(p_2)$ is a decreasing function of p_2 in the interval $p_2 \in [0, 1]$ and we already obtained $f(p_2)|_{p_2=0} > 0$ and $f(p_2)|_{p_2=1} < 0$, there exists one value of p_2 that satisfies $f(p_2) = 0$.

$$\begin{aligned} \frac{d\beta_{p_2}(\lambda_1)}{dp_2} &= \frac{q_{2|\{2\}}p_2}{T}\left\{\frac{c}{(1-\overline{c}\,\overline{p}_2)^2} + \frac{\lambda_1 T}{q_{1|\{1\}}}\frac{\vartheta_2}{(1-\overline{\vartheta}_1 p_2)^2} + \frac{\lambda_1 T}{q_{1|\{1\}}}\frac{-c-\overline{\vartheta}_1\,\overline{c}p_2^2}{(1-\overline{\vartheta}_1 p_2)^2(1-\overline{c}\,\overline{p}_2)^2}\right\} \\ &= \frac{q_{2|\{2\}}p_2}{T}\frac{c(1-\overline{\vartheta}_1 p_2)^2 + \vartheta_2\frac{\lambda_1 T}{q_{1|\{1\}}}(c+\overline{c}p_2)^2 - \frac{\lambda_1 T}{q_{1|\{1\}}}(c+\overline{c}\,\overline{\alpha}p_2^2)}{(1-\overline{\vartheta}_1 p_2)^2(1-\overline{c}\,\overline{p}_2)^2} \\ &= \frac{q_{2|\{2\}}p_2}{T}\frac{f(p_2)}{(1-\overline{\vartheta}_1 p_2)^2(1-\overline{c}\,\overline{p}_2)^2}. \end{aligned} \tag{72}$$

Let p_2° denote the value at which $f(p_2) = 0$. It is found as

$$p_2^\circ = \frac{-c\left(-\overline{\vartheta}_1 + \vartheta_2 \frac{\lambda_1 T}{q_{1|\{1\}}}\overline{c}\right) - \sqrt{c}\sqrt{D}}{c\overline{\vartheta}_1^{\,2} + \vartheta_2 \frac{\lambda_1 T}{q_{1|\{1\}}}(\overline{c}^2 - \overline{c}\,\overline{\vartheta}_1)}, \quad (83)$$

where D is expressed as

$$\begin{aligned} D = c&\left(-\overline{\vartheta}_1 + \vartheta_2 \frac{\lambda_1 T \overline{c}}{q_{1|\{1\}}}\right)^2 \\ &- \left[c\overline{\vartheta}_1^2 + \frac{\lambda_1 T \overline{c}}{q_{1|\{1\}}}\left(\vartheta_2 \overline{c} - \overline{\vartheta}_1\right)\right] \\ &\times \left(1 + (\vartheta_2 c - 1)\frac{\lambda_1 T}{q_{1|\{1\}}}\right). \end{aligned} \quad (84)$$

For $p_2^\circ < p_u < 1$, we can see that p_2° maximizes $\beta_{p_2}(\lambda_1)$. Substituting p_2° into (29) yields the following relation:

$$\begin{aligned} \beta_{p_2}(\lambda_1) \leq &\frac{q_{1|\{1\}}}{T(1-\vartheta_1 c)^2}\left\{\sqrt{c}\sqrt{\frac{\lambda_1 T}{q_{1|\{1\}}}\left(\overline{\vartheta_1 + \vartheta_2 + \vartheta_1\vartheta_2 c}\right)} \right. \\ &\left. - \sqrt{c\overline{\vartheta}_1 + \overline{c}\left(1 - \frac{\lambda_1 T}{q_{1|\{1\}}}\right)}\right\}^2 \triangleq J(\lambda_1) \end{aligned} \quad (85)$$

for $u_1 < \lambda_1 < u_2$. Notice that from $p_2^\circ < p_u$, we can find u_2 as

$$u_2 = \frac{q_{1|\{1\}}}{T} \cdot \frac{\overline{\vartheta_1 + \vartheta_2 + c\vartheta_1\vartheta_2}}{\overline{\vartheta_1 + \vartheta_2 + \vartheta_2 c}}. \quad (86)$$

Finally, at $p_2 = p_u$, referring to the second equation of (38), we have

$$\beta_{p_2}(\lambda_1) \leq \frac{q_{2|\{2\}}}{T}\frac{\vartheta_2}{\overline{\vartheta}_1}\left(1 - \frac{\lambda_1 T}{q_{1|\{1\}}}\right), \quad (87)$$

for $u_2 \leq \lambda_1 < \frac{q_{1|\{1\}}}{T}$.

APPENDIX G DERIVATION OF BACKOFF ALGORITHM

Let $\hat{\mathsf{N}}$ be the random variable for the number of backlogged users at a slot in the system. The joint probability that a slot is found idle and n backlogged users in the system is expressed as

$$\Pr[I, \hat{\mathsf{N}} = n] = \mathfrak{B}_0^n(p)\phi_n(\nu) = \frac{((1-p)\nu)^n}{n!}e^{-\nu}. \quad (88)$$

The marginal probability of an idle slot is found by

$$\Pr[I] = \sum_{n=0}^{\infty} \Pr[I, \hat{\mathsf{N}} = n] = e^{-p\nu}. \quad (89)$$

The *a posteriori* probability that the system has n backlogged users given an idle slot is obtained as

$$\Pr[\hat{\mathsf{N}} = n | I] = \phi_n(\nu(1-p)). \quad (90)$$

By substituting $p = x^*/\nu$ into (90), we have

$$\mathbb{E}[\hat{\mathsf{N}} | I] = \nu - x^*, \quad (91)$$

which implies that we only need to subtract x^* from the previously estimated mean number of backlogged users after an idle backoff slot is observed.

Let us consider that m packets are successfully transmitted.

$$\begin{aligned} \Pr[S = m, \hat{\mathsf{N}} = n] &= \mathfrak{B}_m^n(p)\phi_n(\nu) \\ &= \binom{n}{m} p^m (1-p)^{n-m} \frac{\nu^n}{n!} e^{-\nu}. \end{aligned} \quad (92)$$

As before, we can find the marginal probability of m successful packet transmissions as

$$\Pr[S = m] = \sum_{n=m}^{\infty} \Pr[S = m, \hat{\mathsf{N}} = n] = \frac{(p\nu)^m}{m!}e^{-p\nu}. \quad (93)$$

The *a posteriori* probability for this is found as

$$\begin{aligned} \Pr[\hat{\mathsf{N}} = n | S = m] &= \frac{\Pr[S = m, \hat{\mathsf{N}} = n]}{\Pr[S = m]} \\ &= \frac{((1-p)\nu)^{n-m}}{(n-m)!} e^{-(1-p)\nu}. \end{aligned} \quad (94)$$

After m successful packet transmissions in a slot, the conditional expectation is

$$\mathbb{E}[\hat{\mathsf{N}} | S = m] = \sum_{n=m}^{\infty} n \Pr[\hat{\mathsf{N}} = n | S = m] = (1-p)\nu + m. \quad (95)$$

Using $p = x^*/\nu$, we have

$$\mathbb{E}[\hat{\mathsf{N}} | S = m] = \nu - x^* + m. \quad (96)$$

Here, we need to subtract m_b from (96), which is given in line 3 in Algorithm 1.

When a collision occurs, the joint probability is expressed as

$$\begin{aligned} \Pr[C, \hat{\mathsf{N}} = n] = \Pr[\hat{\mathsf{N}} = n] &- \Pr[I, \hat{\mathsf{N}} = n] \\ &- \sum_{m=1}^{M} \Pr[S = m, \hat{\mathsf{N}} = n]. \end{aligned} \quad (97)$$

From (97), the expectation for $\hat{\mathsf{N}}$ is obtained as

$$\begin{aligned} \mathbb{E}[\hat{\mathsf{N}}, C] &= \mathbb{E}[\hat{\mathsf{N}}] - \mathbb{E}[\hat{\mathsf{N}}, I] - \sum_{m=1}^{M} \mathbb{E}[\hat{\mathsf{N}}, S = m] \\ &= \nu - (1-p)\nu e^{-p\nu} \\ &\quad - \sum_{m=1}^{M}\left(\frac{(\nu p)^m}{(m-1)!} + \frac{\overline{p}\nu(\nu p)^m}{m!}\right)e^{-p\nu} \\ &= \nu - (1-p)\nu \sum_{m=0}^{M} \frac{(\nu p)^m}{m!}e^{-p\nu} - \nu p \sum_{m=0}^{M-1} \frac{(\nu p)^m}{m!}e^{-p\nu} \\ &= \nu\left(1 - \sum_{m=0}^{M} \phi_m(\nu p)\right) + \nu p \frac{(\nu p)^M}{M!} e^{-p\nu}. \end{aligned} \quad (98)$$

Using (89), (93), (97), we have the probability of collision:

$$\Pr[C] = 1 - \Pr[I] - \sum_{m=1}^{M} \Pr[S=m] = 1 - \sum_{m=0}^{M} \frac{(\nu p)^m}{m!} e^{-\nu p} = 1 - \sum_{m=0}^{M} \phi_m(\nu p). \quad (99)$$

Upon a collision, we approximate the resulting distribution of the number of backlogged users by a Poisson distribution with mean $\mathbb{E}[\hat{\mathsf{N}}|C]$:

$$\mathbb{E}[\hat{\mathsf{N}}|C] = \nu + \frac{\nu p\, \phi_M(\nu p)}{1 - e^{-\nu p} - \sum_{i=1}^{M} \phi_i(\nu p)} = \nu + \frac{x^* \phi_M(x^*)}{1 - \sum_{i=0}^{M} \phi_i(x^*)}, \quad (100)$$

where $p = x^*/\nu$ is used. We define constant $\Xi(x^*, M)$ as

$$\Xi(x^*, M) = \frac{x^* \phi_M(x^*)}{1 - \sum_{i=0}^{M} \phi_i(x^*)}. \quad (101)$$

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