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Sky visibility analysis under urban networks: A stochastic geometry approach

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ABSTRACT

We propose a novel framework to analyze a ground user's sky visibility in an urban outdoor network. In order to measure the user's sky visibility, the point process theory is used to represent buildings of urban outdoor networks. We characterize the line-of-sight (LoS) probability between the ground user and a non-terrestrial network node such as a low-Earth-orbit (LEO) satellite. Then, we quantify how many satellites are observable by the user. This provides intuition for cell planning by answering how many satellites are needed for a ground user without discontinuity of network services. Our analysis is cross-validated by numerical experiments.

1. Introduction

Satellite networks have recently emerged as a promising solution to provide universal coverage for seamless networks. Recent research has been focused on LEO satellite networks due to their low latency, low power consumption, and high data rate among satellite networks [1]. In order to optimize network planning and seamless network services, understanding the connectivity between a user on the ground and LEO satellites is essential. However, this connectivity is significantly degraded under a non-line-of-sight (NLoS) environment created by terrestrial blockages between the user and the LEO satellite. Hence, it is essential to investigate the LoS probability of a channel.

Stochastic geometry deals with random spatial patterns [2]. This mathematical tool provides a natural way to define and compute networks' macroscopic properties by modeling network entities' locations using point process theory. Stochastic geometry characterizes the coverage and data rate of networks such as ad-hoc [3] and cellular networks [4]. We will focus on modeling urban outdoor environments, such as building locations and heights, to measure users' LoS visibility in the sky.

1.1. Related works

As the sub-6 GHz spectrum band becomes saturated, many applications use high-frequency bands like mmWave communications. These applications provide stable performance under the LoS environment, which is determined by network geometry. So, understanding the statistical behaviors of blockages and identifying LoS paths is important.

In order to understand these statistical properties, previous works used stochastic geometry [5,6]. The Boolean model was used to determine whether each path is LoS or NLoS [5]. The effect on network performances was studied when the shadowing effect is correlated [6]. Also, the LoS probability of a given path in an urban environment using the Boolean model from point process theory was analyzed [7,8].

Stochastic geometry has gained popularity for modeling non-terrestrial networks, such as satellite networks, due to its ability to provide more tractable coverage analysis compared to traditional grid models, such as Walker's constellation. In this approach, satellite locations are modeled using a homogeneous binomial point process (BPP) on spherical surfaces, as demonstrated in prior studies [9–11]. These studies have established coverage probability by assuming a fixed number of satellites uniformly distributed across the surface of a sphere. The authors in [10] also characterized The distribution of distances for two key links: (1) between the user and the nearest satellite and (2) between satellites located at different altitudes are also characterized [10]. Building on these findings, the downlink coverage probability was derived when a satellite acts as a relay between the user and other satellites [11]. However, these works do not consider the performance of ground users in the LEO satellite networks, which may depend on the terrestrial environment.

1.2. Contributions

This paper proposes a model to analyze the visibility into the sky observed by a typical user in an urban outdoor network. We represent the building locations as a homogeneous marked Poisson point process

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(PPP). We assume that the building heights are independent and identically distributed (i.i.d.) random variables. We set the user's location at the origin and define the visibility into the sky observed by this user. The main contributions are summarized below:

- We propose a novel framework to model the urban outdoor network using a marked point process. The location and mark of points are the location and height of each building, respectively. Based on this model, we analyze the typical user's visibility into the sky. We define the visibility concerning the user's observing angle with respect to the ground and analyze the distribution of the visibility.
- Based on the proposed model, under the general distribution of building heights, we provide analytical formulas for the user's sky visibility as a function of (1) the user's observing angle and (2) the terrestrial network parameters with integral expression. We offer numerous examples to provide intuitions on the behavior of visibility. Further, by using the visibility analysis, we answer how many LEO satellites are observable by the typical user in the numerical experiment section when the satellites are uniformly distributed.

The remainder of this paper is organized as follows. Section 2 describes the system models and defines the typical user's visibility. We analyze the user's visibility in urban outdoor networks in Section 3. Then, we provide the numerical experiments to evaluate the visibility in Section 4. Concluding remarks of this paper are given in Section 5.

2. System model

2.1. Network model

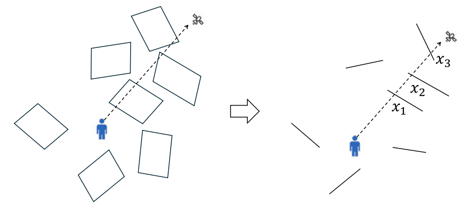
In this part, we first model urban outdoor networks characterized by tall buildings. We model these terrestrial buildings (obstacles) by using the Boolean model of the random shape theory and assume that the shapes of all obstacles are rectangles with random sizes, locations, and orientations as depicted in the left figure of Fig. 1(a). Without loss of generality, we locate the user at the origin. We denote the center of the base of the obstacles as $\Phi^o = \{x_i^o\}_{i\in\mathbb{N}}$[1] which forms a two-dimensional homogeneous PPP with intensity λ^o. Their associated marks, $H = \{h_i\}_{i\in\mathbb{N}}$, $L = \{L_i\}_{i\in\mathbb{N}}$, $\Psi = \{\psi_i\}_{i\in\mathbb{N}}$ represent their heights, lengths and orientations, respectively. These marks are assumed to be i.i.d. and independent of Φ^B. We further assume that the cumulative distribution function (CDF) of H is $F_H(\cdot)$ and Ψ is uniformly distributed in $[0, 2\pi)$.

Since it is hard to fully characterize the visibility in 3D networks as in Fig. 1(a), we focus on visibility analysis along the user's view direction, which is a dotted arrow passing the user in Fig. 1(a) to quantify the user's visibility. The user's view direction is reduced to the model depicted in Fig. 1(b) in which the x-axis is the user's observing direction. The typical user is located at the origin of the x-axis. We define $\Phi = \{x_i\}_{i\in\mathbb{N}}$ as the intersecting points with the user's view direction and terrestrial obstacles. Then, Φ becomes an one-dimensional homogeneous PPP with intensity $\lambda = \frac{2}{\pi}\lambda^o\mathbb{E}[L]$ [5].

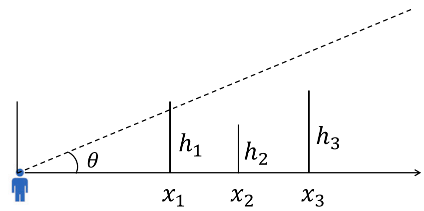
2.2. Visibility

Let θ be the user's observing angle with respect to the ground (x-axis) as depicted in Fig. 1(b). In other words, θ is the angle formed by the ground and the user's view direction. Further, we define $N(\theta)$ as the number of buildings that block the user's view direction when the observing angle is θ. For example, in Fig. 1(b), $N(\theta) = 1$. If $N(\theta) = 0$, the path observed by the user with observing angle θ is LoS. We define $\theta_{\min}$ as the minimum θ which satisfies $N(\theta) = 0$. In other words, $\theta_{\min}$ is the minimum angle for the LoS path where $\tan\theta_{\min} = \max_i \frac{h_i}{x_i}$.

[1] $\mathbb{N}$ is the set of natural numbers.



(a) Urban outdoor network. (Left: 3D view, Right: Projection view)



(b) Reduced 2D model focused on the user's view direction.

Fig. 1. Network model.

3. Visibility analysis

In this section, we analyze the user's sky visibility by providing the distribution of $N(\theta)$.

Theorem 1. *$N(\theta)$ is a Poisson random variable with parameter $\lambda\int_0^\infty 1 - F_H(x\tan\theta)dx$.*

Proof. Let Φ_B be the subset of Φ where the tangent of θ is smaller than $\frac{h_i}{x_i}$. Then, by the thinning theorem of PPP [2], we can obtain that Φ_B is a Poisson point process with intensity measure $\lambda\int_0^\infty p(x)dx$ where

$$\begin{aligned} p(x) &= \mathbb{P}\left[\frac{h}{x} > \tan\theta\right] \\ &= \mathbb{P}[h > x\tan\theta] \\ &= 1 - F_H(x\tan\theta), \end{aligned} \tag{1}$$

which is a retention function. So, $N(\theta)$, which is the number of points in Φ_B, is a Poisson random variable with parameter $\lambda\int_0^\infty 1-F_H(x\tan\theta)dx$.

Remark 1. We can also obtain the same result with (1) by using the max shot noise analysis. Please see [12].

Based on Theorem 1, we provide an analytical expression of the probability of LoS visibility by observing angle θ in the following corollary.

Corollary 1. *The LoS probability of the user's sky visibility with the observing angle θ is*

$$\mathbb{P}[N(\theta) = 0] = \exp\left(-\lambda\int_0^\infty 1 - F_H(x\tan\theta)dx\right), \tag{2}$$

for $\theta \in [0, \frac{\pi}{2})$.

According to [13], the LoS probability based on the elevation angle is described from an average perspective in urban environments. However, it does not provide detailed information about specific urban environments. Corollary 1 describes how the distribution of LoS probability varies depending on network parameters in different urban environments. We provide an LoS probability with some explicit examples of building height distribution.

Example 1. When the building heights follow a Weibull distribution with the shape parameter $k > 0$ and the scale parameter $\mu > 0$, the CDF of height, $F_H(x)$ is

$$F_H(x) = 1 - e^{-\left(\frac{x}{\mu}\right)^k}. \tag{3}$$

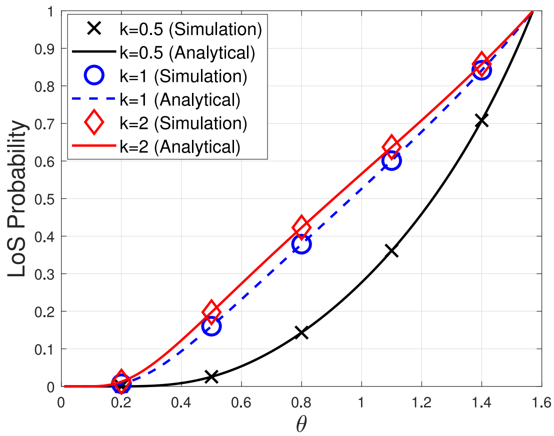


Fig. 2. LoS probability with different shape parameters.

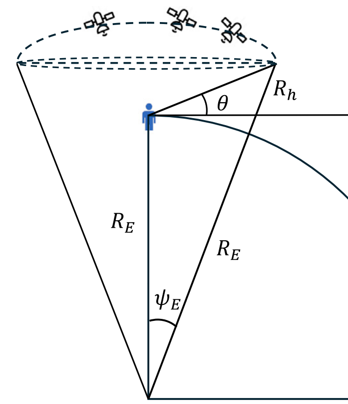


Fig. 3. Observable spherical cap by the typical user.

Weibull distribution is widely used to model a broad range of random variables by adjusting the shape and scale parameters. When $k = 1$ and $k = 2$, the Weibull distribution reduces to the exponential distribution and the Rayleigh distribution, respectively. For $k < 1$, the Weibull distribution becomes a heavy-tailed distribution. By plugging (3) into (2), the LoS probability becomes

$$\mathbb{P}[N(\theta) = 0] = \exp\left(-\frac{\lambda}{\mu \tan\theta}\Gamma\left(1+\frac{1}{k}\right)\right). \tag{4}$$

Fig. 2 illustrates the LoS probability under $\lambda = 1$ and $\mu = 1$ with different k. As the observing angle θ increases, the LoS probability also increases. We can also observe that larger shape parameter k leads to a higher LoS probability.

Corollary 2. *The mean number of buildings that block the user's view direction with observing angle θ is*

$$\mathbb{E}[N(\theta)] = \lambda \int_0^\infty 1 - F_H(x\tan\theta)dx, \tag{5}$$

since $N(\theta)$ follows Poisson distribution.

Corollary 3. *The probability density function (PDF) of $\theta_{\min}$ is*

$$f_{\theta_{\min}}(\theta) = \frac{d}{d\theta}\exp\left(-\lambda\int_0^\infty 1 - F_H(x\tan\theta)dx\right). \tag{6}$$

Proof. By definition, the CDF of $\theta_{\min}$ is

$$\mathbb{P}[\theta_{\min} < \theta] = \mathbb{P}\left[\max_i \frac{h_i}{x_i} < \tan\theta\right] = \mathbb{P}[N(\theta) = 0]. \tag{7}$$

So, the PDF of $\theta_{\min}$ is simply obtained by differentiating $\mathbb{P}[N(\theta) = 0]$.

Example 2. As in Example 1, let us assume that the building height follows a Weibull distribution with the shape parameter $k > 0$ and the scale parameter $\mu > 0$. Then, the PDF of $\theta_{\min}(\theta)$ is

$$f_{\theta_{\min}}(\theta) = \exp\left(-\frac{\lambda\mu}{\tan\theta}\Gamma\left(1+\frac{1}{k}\right)\right)\frac{\lambda\mu}{\cos^2\theta\tan^2\theta}\Gamma\left(1+\frac{1}{k}\right). \tag{8}$$

4. Numerical experiments and discussions

4.1. How many satellites are observable by the typical user?

In this part, we discuss how many LEO satellites are observable by the typical user on the ground.

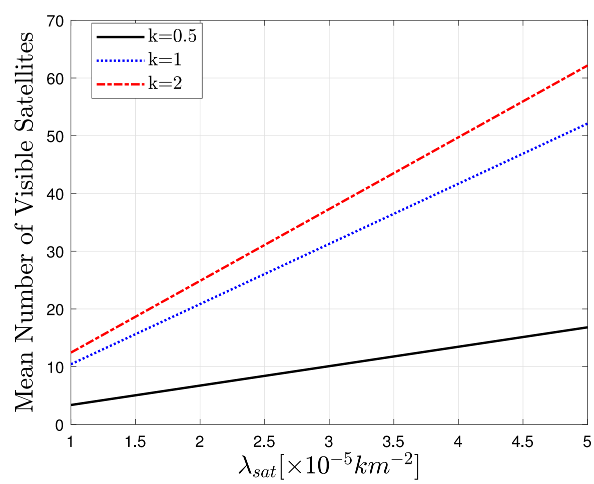


Fig. 4. Mean number of the observable satellites.

Fig. 3 illustrates the projection view of the network geometry of Earth, satellites, and the ground user. Let R_E, R_h be Earth's radius and the satellites' altitude. Without loss of generality, we locate the center of Earth and the typical user at $(0, 0, 0)$ and $(0, 0, R_E)$ in the three-dimensional Cartesian coordinate. Let ψ_E be the angle formed by the user, the center of Earth, and the intersection of a sphere with radius $R_E + R_h$ centered at $(0, 0, 0)$ and user's observing path with observing angle θ. Then, the relation of θ and ψ_E becomes

$$\psi_E = \arccos\left(\frac{1}{R_E+R_h}\left(R_E\cos^2\theta + \sin\theta\sqrt{R_E^2\sin^2\theta + R_h^2 + 2R_ER_h}\right)\right). \tag{9}$$

So, the mean area of the visible spherical cap by the ground user is

$$\mathbb{E}\left[2\pi(R_E+R_h)^2(1-\cos\psi_E)\right] = 2\pi(R_E+R_h)^2\int_0^{\frac{\pi}{2}}\left(1 - \frac{1}{R_E+R_h}\left(R_E\cos^2\theta + \sin\theta\sqrt{R_E^2\sin^2\theta + R_h^2 + 2R_ER_h}\right)\right) \times f_{\theta_{\min}}(\theta)d\theta. \tag{10}$$

Fig. 4 illustrates how many LEO satellites are observable by the typical user with terrestrial buildings. λ_{sat} is defined as the average number of satellites observed by the user without terrestrial blockages when R_E and R_h are 6371 (km) and 500 (km), respectively. We assume that $\lambda = 1$ and building heights are distributed following the exponential distribution with parameter $\mu = 1$. From Fig. 4, we can check that as λ_{sat} increases and the shape parameter k increases, a heavier tail distribution of building height results in an increase in the number of visible LEO satellites by the ground user.

5. Conclusion

In this paper, we proposed a novel framework to investigate the sky visibility of a ground user in urban outdoor networks by using point process theory and stochastic geometry. Under that model, we analyzed the distribution of sky visibility as a function of network parameters. We quantify the LoS probability of channels between the ground user and LEO satellites through the analytical results, and provided numerical experiments by examining how many satellites are observable by the typical user.

As a potential direction for further research, our model can be extended to a 3D model that more closely represents real-world environments or used for analyzing connectivity or coverage between ground users and non-terrestrial network nodes.

CRediT authorship contribution statement

Heejung Yu: Methodology, Conceptualization. **Sooyeob Jung:** Writing – review & editing, Conceptualization. **Joon Gyu Ryu:** Writing – original draft, Project administration, Investigation, Conceptualization. **Junse Lee:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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