Microarticle

# A real-time microwave imaging of unknown anomaly with and without diagonal elements of scattering matrix 

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#### Abstract

In this contribution, a real-time non-iterative microwave imaging of unknown anomaly from scattering matrix whose elements are scattering parameters measured at the dipole antennas considered. For this, we apply Kirchhoff migration imaging technique when the diagonal elements of scattering matrix are available or unavailable, and confirm that opposite to the traditional researches, one can obtain good result without diagonal elements. Simulation results with synthetic data are presented to support our confirmation.


## Introduction

The real-time identification of the outline shapes or locations of unknown anomalies from scattering matrix is an interesting inverse scattering problem closely related to the microwave imaging. Various real-time imaging techniques have already been investigated and Kirchhoff migration (KM) has been confirmed as a fast, stable, and effective imaging technique (see [1-3]). To apply the KM, complete elements of the scattering matrix must be collected. However, in realworld applications, it is very hard to measure the diagonal elements of a scattering matrix [4-6]. Thus, traditional Kirchhoff migration would not make sense for real-world microwave imaging.

In this paper, we consider a microwave imaging performed by a Kirchhoff migration-based technique when the diagonal elements of a scattering matrix could or could not be measured. For this purpose, we introduce a new scattering matrix by exchanging the diagonal elements to the zero and applying this matrix to imaging. Simulation results are presented here to ascertain the feasibility, compare the imaging performances, and confirm the good imaging results of the new scattering matrix.

## Scattering parameter and imaging function

Assume that an anomaly $D$ is to be imaged and is located at $\mathbf{r}_{\star}$ in a region of interest (ROI) $\Omega . D$ is surrounded by $N$-different dipole antennas located at $\mathbf{a}_{n}, n=1,2, \cdots, N$, and characterized by dielectric
permittivity and electrical conductivity at a given angular frequency $\omega=2 \pi f$. We set the magnetic permeability at every $\mathbf{r} \in \Omega$ as a constant, $\mu(\mathbf{r}) \equiv \mu_{\mathrm{b}}=\mu_{0}=1.256 \times 10^{-6} \mathrm{H} / \mathrm{m}, \quad \varepsilon_{\mathrm{b}}=\varepsilon_{\mathrm{rb}} \cdot \varepsilon_{0}$ and $\varepsilon_{\star}=\varepsilon_{\mathrm{r} \star} \cdot \varepsilon_{0}$ denote the permittivity of $\Omega$ and $D$, respectively. Here, $\varepsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ is the vacuum permittivity. Conductivities $\sigma_{\mathrm{b}}$ and $\sigma_{\star}$ are defined analogously. In addition, the wavenumber $k$ is given by $k^{2}=\omega^{2} \mu_{\mathrm{b}}\left(\varepsilon_{\mathrm{b}}+i \sigma_{\mathrm{b}} / \omega\right)$.

Let $\mathbf{E}_{\text {tot }}\left(\mathbf{r}, \mathbf{a}_{m}\right)$ be the total electric field resulting from the existence of the anomaly $D$ measured at the $\mathbf{a}_{m}$. Similarly, we denote $\mathbf{E}_{\text {inc }}\left(\mathbf{a}_{n}, \mathbf{r}\right)$ be the incident field resulting from the point current density at the $\mathbf{a}_{n}$. We denote $\mathrm{S}(n, m)$ be the scattered field $S$-parameter (or scattering parameter) with a transmitter $n$ and receiver $m$ that are obtained by subtracting the $S$-parameters with and without anomalies, respectively.

We now assume that $D$ is a "small" anomaly. Then, based on the researches [4,5,7] and application of the Born approximation, $\mathrm{S}(n, m)$ is given by

$$
\begin{align*}
\mathrm{S}(n, m) & =\frac{\mathrm{i} k^{2}}{4 \omega \mu_{\mathrm{b}}} \int_{\Omega} \chi(\mathbf{r}) \mathrm{E}_{\mathrm{inc}}\left(\mathbf{a}_{n}, \mathbf{r}\right) \cdot \mathbf{E}_{\mathrm{tot}}\left(\mathbf{r}, \mathbf{a}_{m}\right) \mathrm{d} \mathbf{r} \\
& \approx \frac{\mathrm{ik}{ }^{2} \operatorname{area}(D)}{4 \omega \mu_{\mathrm{b}}} \int_{\Omega} \chi(\mathbf{r}) \mathrm{E}_{\mathrm{inc}}^{(\mathrm{z})}\left(\mathbf{a}_{n}, \mathbf{r}_{\star}\right) \mathrm{E}_{\mathrm{inc}}^{(z)}\left(\mathbf{a}_{m}, \mathbf{r}_{\star}\right) \tag{1}
\end{align*}
$$

where $\mathrm{E}_{\text {inc }}^{(z)}$ is the $z$-component of $\mathbf{E}_{\text {inc }}$ and
$\chi\left(\mathbf{r}_{\star}\right)=\frac{\varepsilon_{\star}-\varepsilon_{\mathrm{b}}}{\varepsilon_{\mathrm{b}}}+i \frac{\sigma_{\star}-\sigma_{\mathrm{b}}}{\omega \varepsilon_{\mathrm{b}}}$.
Let K be a scattering matrix whose elements are $\mathrm{S}(n, m)$ :

[^0]

Fig. 1. Maps of $f_{\mathrm{FM}}(\mathbf{r})$ (left) and $f_{\mathrm{DM}}(\mathbf{r})$ (right) when $D$ is a small anomaly.
$\mathrm{K}=\left[\begin{array}{ccccc}\mathrm{S}(1,1) & \mathrm{S}(1,2) & \cdots & \mathrm{S}(1, N-1) & \mathrm{S}(1, N) \\ \mathrm{S}(2,1) & \mathrm{S}(2,2) & \cdots & \mathrm{S}(2, N-1) & \mathrm{S}(2, N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathrm{S}(N, 1) & \mathrm{S}(N, 2) & \cdots & \mathrm{S}(N, N-1) & \mathrm{S}(N, N)\end{array}\right]$.
From the approximation (1), let us introduce a test vector
$\mathbf{W}(\mathbf{r})=\left(\mathrm{E}_{\text {inc }}^{(z)}\left(\mathbf{a}_{1}, \mathbf{r}\right), \mathrm{E}_{\text {inc }}^{(z)}\left(\mathbf{a}_{2}, \mathbf{r}\right), \cdots, \mathrm{E}_{\text {inc }}^{(z)}\left(\mathbf{a}_{N}, \mathbf{r}\right)\right)$.
Then, the traditional Kirchhoff migration-based imaging function can be introduced as follows (see [1,2] for instance):
$f_{\mathrm{FM}}(\mathbf{r})=\left|\overline{\mathbf{W}}(\mathbf{r}) \mathbb{K} \overline{\mathbf{W}}(\mathbf{r})^{T}\right|, \quad \mathbf{r} \in \Omega$.
On the basis of [1-3], the map of $f_{\mathrm{FM}}(\mathbf{r})$ will contain a peak of large magnitude at $\mathbf{r}=\mathbf{r}_{\star} \in D$ and one of small magnitude at $\mathbf{r} \in \Omega \backslash \bar{D}$. However, unlike the conventional simulation setup, each of the $N$ antennas is used for signal transmission, whereas the remaining $N-1$ antennas are used for signal reception $[5,6]$ so that the diagonal elements of $K$ are unknown. Due to this reason, we introduce a new scattering matrix D
$\mathrm{D}=\left[\begin{array}{ccccc}0 & \mathrm{~S}(1,2) & \cdots & \mathrm{S}(1, N-1) & \mathrm{S}(1, N) \\ \mathrm{S}(2,1) & 0 & \cdots & \mathrm{~S}(2, N-1) & \mathrm{S}(2, N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathrm{S}(N, 1) & \mathrm{S}(N, 2) & \cdots & \mathrm{S}(N, N-1) & 0\end{array}\right]$
and a new imaging function:
$f_{\mathrm{DM}}(\mathbf{r})=\left|\overline{\mathbf{W}}(\mathbf{r}) \mathrm{D} \overline{\mathbf{W}}(\mathbf{r})^{T}\right|, \quad \mathbf{r} \in \Omega$.
Fortunately, it is possible to obtain good imaging results via the mapping of $f_{\mathrm{DM}}(\mathbf{r})$.

Remark If $D$ is a "large" anomaly, the approximation (1) is not valid because the Born approximation cannot be applied. Thus, the vector $\mathbf{W}(\mathbf{r})$ of (2) must be defined in a different form. Due to this reason, it will be very hard to identify the outline shape of $D$ via the map of $f_{\mathrm{FM}}(\mathbf{r})$ and $f_{\mathrm{DM}}(\mathbf{r})$.

## Simulation results

For performing numerical simulation, $N=16$ dipole antennas equally distributed on a circle with radius of 0.090 m centered at the origin. The parameters were set to $\varepsilon_{\mathrm{rb}}=20$ and $\sigma_{\mathrm{b}}=0.2 \mathrm{~S} / \mathrm{m}$ at $f=1.2 \mathrm{GHz}$. Notice that $\mathrm{S}(n, m)$ and $\mathrm{E}_{\mathrm{inc}}^{(z)}\left(\mathbf{a}_{n}, \mathbf{r}\right)$ were generated through the CST STUDIO SUITE.

Fig. 1 shows the maps of $f_{\mathrm{FM}}(\mathbf{r})$ and $f_{\mathrm{DM}}(\mathbf{r})$ when the anomaly $D$ is a small circle with $\varepsilon_{r \star}=55, \sigma_{\star}=1.2 \mathrm{~S} / \mathrm{m}, \alpha=0.01 \mathrm{~m}=0.18 \lambda_{\mathrm{b}}$, and $\mathbf{r}_{\star}=(0.01 \mathrm{~m}, 0.03 \mathrm{~m})$. Here, $\lambda_{\mathrm{b}}$ denotes the wavelength in the background. The results indicate that it is possible to recognize the location and outline shape of $D$ via the maps of $f_{\mathrm{FM}}(\mathbf{r})$ and $f_{\mathrm{DM}}(\mathbf{r})$. It is interesting to observe that the map of $f_{\mathrm{FM}}(\mathbf{r})$ contains several peaks of large magnitude and artifacts that disturb the identification of any anomaly while the map of $f_{\mathrm{DM}}(\mathbf{r})$ contains much fewer artifacts that can be


Fig. 2. Maps of $f_{\mathrm{FM}}(\mathbf{r})$ (left) and $f_{\mathrm{DM}}(\mathbf{r})$ (right) when $D$ is a large anomaly.
disregarded.
We now consider the imaging of a large anomaly. For this, we select an anomaly $D$ as a circle with $\varepsilon_{\mathrm{r} \star}=15, \sigma_{\star}=0.5 \mathrm{~S} / \mathrm{m}$, $\alpha=0.05 \mathrm{~m}=0.90 \lambda_{\mathrm{b}}$, and $\mathbf{r}_{\star}=(0.01 \mathrm{~m}, 0.02 \mathrm{~m})$. As we discussed in the Remark, in contrast to the imaging of a small anomaly, it is hard to identify the outline shape of $D$ through the map of $f_{\mathrm{FM}}(\mathbf{r})$ because of the appearance of the artifacts, refer to Fig. 2. It is interesting to observe that surprisingly, it is possible to identify $D$ through the map of $f_{\mathrm{DM}}(\mathbf{r})$. Thus, we can examine that making the diagonal elements of the scattering matrix to zero successfully improves the imaging performance.

## Conclusion

We considered the Kirchhoff migration-based imaging function for a real-time imaging of unknown anomalies when the diagonal elements of scattering matrices could or could not be handled. Unlike the traditional studies, we examined that it was possible to apply Kirchhoff migration by making the diagonal elements of the scattering matrix to zero and found that doing so yielded better results than with the diagonal elements. An additional analysis of the imaging function to support examined phenomenon will be conducted for a forthcoming work.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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