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A More Configurable Finite Memory Event Triggering Control Scheme for Networked Control Systems

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ABSTRACT For a networked control system (NCS), this article proposes a more configurable event triggering control scheme that employs a finite memory structure with tuning parameters for efficiently utilizing network resources in transient and steady-state responses. The proposed finite memory event triggering control scheme (FMETS) uses the current and past states on the recent finite time horizon to asymptotically stabilize a closed-loop system. Tuning parameters can be configured according to the forgetting level, the triggering threshold, and the memory size. Since the FMETS takes a sample-and-hold approach, the undesirable Zeno effect can be avoided automatically. To use an efficient computation solver, the proposed FMETS is formulated into linear matrix inequalities (LMIs). It is shown through a simulation study that the proposed FMETS provides less conservative results and furthermore enables effective configuration for high control performance and efficient network resource utilization.

INDEX TERMS Networked control system (NCS), event triggering control scheme (ETS), event triggering condition, finite memory.

I. INTRODUCTION

Networked control systems (NCS) involve communication networks to connect subsystems such as sensors, actuators, and controllers, and then construct a closed-loop system [1]–[4]. For communication networks, wired and wireless networks are properly designed in terms of cost, feasibility, and other technical details, according to which the corresponding NCSs and their stability should be analyzed [4], [5]. Recently, NCSs have been widely applied to various communication networks such as CAN, Ethernet, Wifi and so on, in order to be applied for autonomous driving [6]–[8], unmanned aerial vehicle (UAV) [9], and multi-agent system [10], [11]. However, such useful NCSs often suffer from network-induced problems including transmission delay, signal disorder, packet loss, and limited network resources [12]–[15]. Much attention has been paid to research on the aforementioned problems.

As one of the ways to make the best use of limited network resources, event triggering control scheme (ETSs) have been proposed [16], [17]. Generally, ETSs employ two strategies: event triggering communication and the zero-order-hold (ZOH) sampling. Event triggering conditions are designed to determine whether to transmit the control input to an actuator over a communication network. *i.e.*, if event triggering conditions are met or triggered, the control input is updated. The ZOH sampling is employed to hold the control input value until the next event is triggered. With these two strategies, ETSs can prevent inefficient use of network resources. Such useful ETSs have been widely applied to various systems

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including linear systems [17]–[19], nonlinear systems [20]–[24], multi-agent systems [25]–[29], Markov jump systems [30]–[32], and complex dynamical networks [33]–[35]. For practical applicability, ETSs are favorable to discrete-time systems [36]. For continuous-time systems, ETSs have been tried to be discretized in order to avoid the Zeno effect that the infinite number of triggering occurs during the finite time interval [37]–[39].

Recently, much attention has been paid to research on designing appropriate event triggering conditions in ETS. As a simple approach, the absolute event triggering control scheme (AETS) has been investigated for continuous-time systems. Its prescribed threshold value is compared with the measured state and then triggering occurs if the latter is greater than the former. Though the AETS has advantages of ensuring input to state stability and using a simple structure, it lacks flexibility and hence may achieve conservative results [40]. For more general structure, the relative event triggering control scheme (RETS) was proposed, which compares the current state with its difference from the latest triggered state [41]. It means that the current state plays the role of a threshold for triggering. The RETS has been widely studied with various types of controllers: state feedback, output feedback [42], [43], and observer-based control [44]–[46]. However, such useful RETSs have the limitations of lacking tuning parameters, and waste the network resources unintentionally by using only the current states for event triggering conditions. Specially, the network resource is wasted in the steady-state response due to its structural characteristics of the RETSs. To solve this problem, the integral event triggering control scheme (IETS) has been proposed, but it requires a large amount of memory for the integral calculation, which serves as another cost. IETS still lacks tuning parameters due to use of a simple integral, and the minimum inter-event time can not be guaranteed, which can cause the Zeno effect in a continuous-time system [47]–[49]. Recently, studies on event triggering control schemes employing the fixed memory have been proposed. However, these have aimed to improve H_∞ performance [50] or to reduce the crest or trough of state/output response [51].

To address these problems, a more configurable finite memory event triggering control scheme (FMETS) is proposed for NCSs. First, a novel event triggering condition is proposed by employing a finite memory strategy, which stores both the current and past states on the recent finite time horizon. As in the conventional RETSs, the proposed event triggering condition reflects the previous triggering results as residuals. Since these residuals play roles of additional margins, the event triggering condition is less conservative, and the proposed FMETS can further save the network resource. Tuning parameters are also configured according to the forgetting level, the triggering threshold, and the memory size to make the better use of network resources in transient and steady-state responses. It is shown through simulation studies that the effects of parameters on transient and steady-state responses are numerically illustrated and the proposed FMETS has less conservative results with proper tuning parameters while overcoming the limitations of the conventional RETS and IETS. Second, since the FMETS takes a sample-and-hold approach, the undesirable Zeno effect can be avoided automatically in continuous-time systems. A sampler is employed to avoid the Zeno effect in continuous-time systems, but the conventional IETS can not be used with the sampler due to structural problems. For efficient computation, an event triggering condition and a state feedback controller of the FMETS are simultaneously formulated into linear matrix inequalities (LMIs).

This article is organized as follows: In Section II, an NCS and the conventional RETS are briefly introduced and preliminaries are are provided for better understanding. The FMETS is proposed in Section III, and its stability analysis is given in Section IV. In Section V, three numerical examples are provided to illustrate the effectiveness of the FMETS. Finally, a conclusion is drawn in Section VI.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this section, we introduce a linear continuous-time system with the sampler, its discretized model, and the conventional RETS. Let us consider the following linear continuous-time systems

$$\dot{x}(t) = A_c x(t) + B_c u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, t, A_c and B_c are the states, the control input, the time instance, the system matrix, and input matrix with appropriate dimensions respectively. The pair (A_c, B_c) of the system is assumed to be controllable. If the sampler with the sampling time h is applied to (1), a discretized model is obtained from [12] as follows:

$$x(k+1) = Ax(k) + Bu(k) \tag{2}$$

where k is the discrete time point and $x(k)$ and $u(k)$ are the states and the control input updated every h steps, respectively. The pair (A, B) is assumed to be controllable and is given as

$$A = e^{A_c h}, \quad B = \int_0^h e^{A_c s} B_c ds.$$

In this article, we ensure the stability of a continuous-time system with the sampler by using a discretized model, which can be easily extended in a discrete-time system. Then, a state feedback controller in the RETS is presented as

$$u(k) = Kx(k_s), \quad k \in [\, k_s, k_{s+1} \,) \tag{3}$$

where K is the state feedback control gain matrix to be determined later on, k_s is the latest triggering instant, and k_{s+1} is the next triggering instant. The conventional relative event triggering condition is given as

$$e^T(k)\Omega e(k) \le \rho x^T(k)\Omega x(k) \tag{4}$$

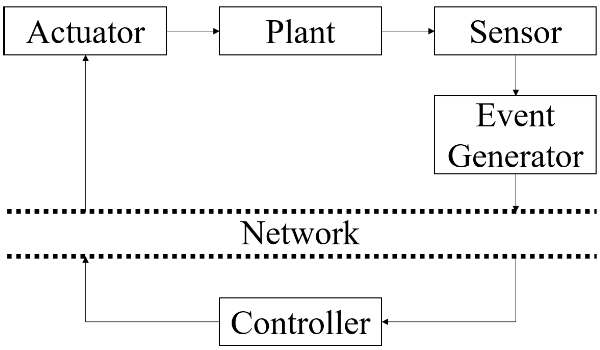
where $e(k)$ is the difference between the latest triggering state and the current state *i.e.*, $e(k) = x(k_s) - x(k)$, Ω is a positive definite weighting matrix and $\rho \in (0, 1)$ is a triggering threshold. The update time of data considering (4) is described as

$$k_{s+1} = \min_{k}\{k > k_s | e^T(k)\Omega e^T(k) \geq \rho x^T(k)\Omega x(k)\} \quad (5)$$

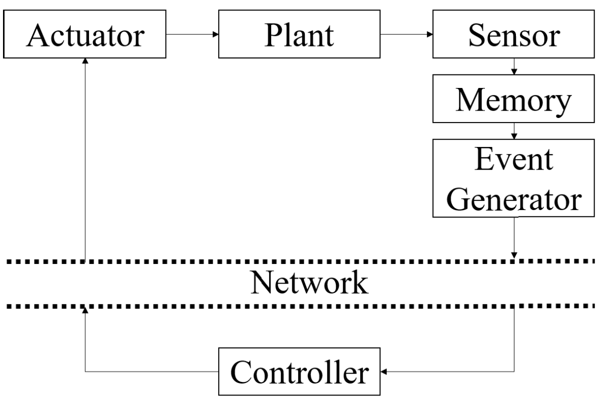
where k_s and k_{s+1} are the sth triggered instant and the next $(s+1)$th triggered instant as in (3). $\Phi_s = [\,k_s,\ k_{s+1})$ is presented as the holding interval of controller (3). Then, the control gain is rewritten as

$$u(k) = Kx(k) + Ke(k), \quad k \in \Phi_s. \quad (6)$$

The whole description of NCS is detailed in Figure 1(a).



(a) Conventional NCS with the ETS.



(b) NCS with the proposed FMETS.

FIGURE 1. The structure of an NCS model.

III. FINITE MEMORY EVENT TRIGGERING CONTROL SCHEME

In this section, the FMETS is proposed through three subsections. The first subsection describes the state feedback controller used in the FMETS. A new event triggering condition using the finite memory is proposed in the second subsection. The effect of tuning parameters on the proposed event triggering condition is analyzed in the last subsection.

A. STATE FEEDBACK CONTROLLER

The proposed FMETS employs a state-feedback controller updated by event triggering condition as in (3).

$$u(k) = Kx(k_s), \quad k \in [\,k_s, k_{s+1}\,)$$

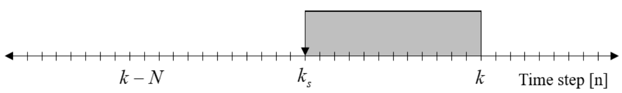
where the state feedback control gain K is determined to guarantee the stability of the system, k_s is the latest triggering instant. The control input $u(k)$ is held until the next triggering instant k_{s+1}.

B. FINITE MEMORY EVENT TRIGGERING CONDITION

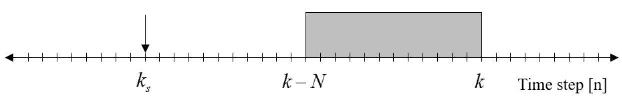
To propose a novel event triggering condition, we employ the finite memory structure. The structure of the NCS is shown in Figure 1(b). The finite memory stores the states of the system. However, since the memory size is finite, the storage method is implemented in a stack format. That is, whenever the state of the system is updated, the state value is accumulated in the memory and when the capacity of the memory becomes full and a new state is updated, the oldest state is discarded and the new state is stored. As a result, the finite stacked memory stores both current and past states on the recent finite time horizon. Using the states of the system stored in the finite memory, we propose a novel event triggering condition as follows:

$$\sum_{i=0}^{N_s} \gamma^i e^T(k-i)\Omega e(k-i) \leq \sum_{i=0}^{N_s} \gamma^i x^T(k-i)\Omega_\rho x(k-i) \quad (7)$$

where k is the current time step, $e(k-i)$ is the difference between triggering states and delayed states *i.e.*, $e(k-i) = x(k_s) - x(k-i)$, ρ is a triggering threshold, $\gamma \in [0, \infty)$ is a forgetting level, and Ω is a positive definite weighting matrix, $\Omega_\rho \triangleq \rho\Omega$. It is noted that N_s is the minimum between the maximum number of available past states N and $k - k_s$ which means how much time elapses since the latest triggering instant, *i.e.*, $N_s \triangleq \min(N, k - k_s)$. The time interval associated with the proposed event triggering condition is illustrated in Figure 2. The analysis of tuning parameters for the proposed event triggering condition is described in next subsection.



(a) $k_s \geq k - N$.



(b) $k_s < k - N$.

FIGURE 2. The time interval associated with the proposed event triggering condition.

C. THE ANALYSIS OF TUNING PARAMTERS FOR THE FMETS

In this subsection, the analysis of tuning parameters for the FMETS is provided. To analyze the effect of N, we define a event triggering residual ε_k as follows:

$$\varepsilon_k \triangleq \rho x^T(k)\Omega x(k) - e^T(k)\Omega e(k). \tag{8}$$

By using (8), we can represent (4) and (7) as

$$\varepsilon_k \geq 0,$$
$$\sum_{i=0}^{N_s} \gamma^i \varepsilon_{k-i} \geq 0.$$

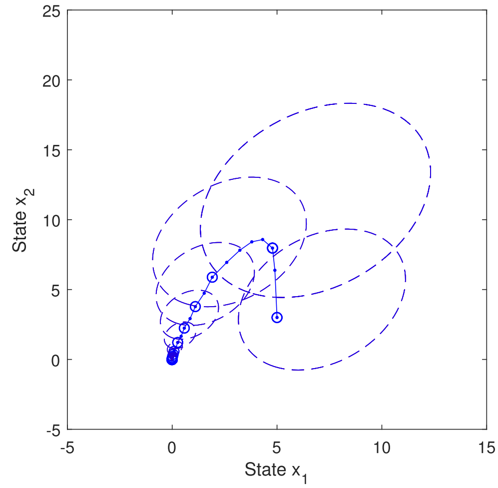
For convenience of analysis, we represent the event triggering condition (4) as a quadratic inequality for $x(k)$ as follows:

$$(x(k) - \frac{1}{1-\rho}x(k_s))^T \Omega (x(k) - \frac{1}{1-\rho}x(k_s)) \leq \frac{\rho}{(1-\rho)^2}x(k_s)^T \Omega x(k_s). \tag{9}$$

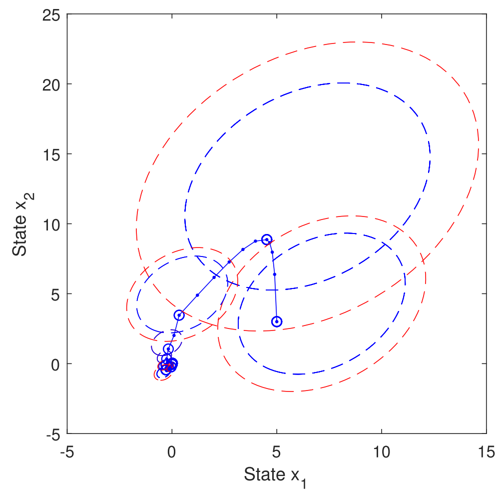
Since Ω is positive definite and $\rho \in (0, 1)$, the inequality (9) means the inner region of an n-dimensional ellipsoid, which is called a triggering region. In other words, the control input is kept as long as a state x stays in the triggering region and if a state escapes from it, the control input is updated and the new triggering region is generated. In this point of view, ε_k means how close a state is to the center of the triggering region in the k step. The proposed event triggering condition consists of the sum of the relative event triggering conditions in the N_s interval. It is a combined version of the current event triggering inequalities and the event triggering residuals as follows:

$$(x(k) - \frac{1}{1-\rho}x(k_s))^T \Omega (x(k) - \frac{1}{1-\rho}x(k_s)) \leq \frac{\rho}{(1-\rho)^2}x(k_s)^T \Omega x(k_s) + \sum_{i=1}^{N_s} \gamma^i \varepsilon_{k-i}. \tag{10}$$

Physically, the event triggering residuals of the previous step enlarge the size of the triggering region. In other words, they play the role of reducing the possibility to trigger. Figure 3 illustrates the effect of the proposed event triggering condition. Figure 3(a) and Figure 3(b) show the triggering regions of the RETS and the FMETS with $N = 4$ and $\gamma = 1$, respectively. To analyze the physical meaning of the proposed event triggering condition and show the effect of the memory size N, Figure 3(a) and 3(b) have the same values for the triggering threshold ρ, the event triggering matrix Ω, the control gain K, and the initial state x_0. In Figure 3, "·" means state trajectories through every time step and "⊙" means triggering states. The blue ellipsoid represents triggering regions in the RETS and the red ellipsoid shows triggering regions generated when the triggering condition is met in the FMETS. In Figure 3(a), states move by the control input in the triggering region and if the state leaves this region, the triggering occurs. On the other hand, in Figure 3(b), by the effect of an event triggering residuals, the triggering region is enlarged and larger than that in the RETS because event triggering residuals keep a positive value in a blue RETS triggering region. Therefore, there are some state regions where the triggering occurs in RETS but not in the FMETS. As a result, the proposed event triggering condition alleviates the conventional relative event triggering condition because past event triggering residuals have a weakening effect on the relative event triggering condition and guarantee that the triggering number is less than or equal to that of the RETS. In this numerical example, the triggering number of Figure 3(a) and Figure 3(b) are 61 and 36 in 120 steps, respectively.



(a) State transitions using the RETS.



(b) State transitions using the FMETS.

FIGURE 3. Triggering boundaries generated from the event triggering conditions.

ρ is the triggering threshold that has a value $\in (0, 1)$. Since ρ changes the size of the triggering region in (9) and (10) proportionally, it is effective in the transient response where the size of the latest triggering region is large relatively. $\gamma \in [0, \infty)$ is the forgetting level that represents how much the past data is reflected in the event triggering condition. For example, if $\gamma = 1$, it means that the past state and the current state have an same effect on the event triggering condition, and the closer γ value is to 0, the more important the current state is considered than past one. On the contrary, if γ has a value larger than 1, it means that the effect of the past states is particularly considered. Interestingly, due to the structure of the ETS, the residuals farther from the current time have large values, and it can enlarge the triggering region. The characteristics of each tuning parameter are covered in more detail in Section V.

Remark 1: There are additional physical benefits of the proposed event triggering condition (7), although they are not considered in this article. First, the practical implementation is easy. Because of the concept of the stack that discards the old data and adds the new data, it is composed of simple addition and subtraction. Second, the finite stack structure can reduce the effect of the disturbance or the noise on the past data relatively. In the case of the integral ETS, the past data with the disturbance or the noise degrades the performance of the event triggering. However, the proposed event triggering condition (7) can discard the past data with the disturbance or the noise.

IV. MAIN RESULTS

In this section, it is described as the stability and the design of the FMETS for a discretized system.

A. LEMMA AND NOTATION

First, a useful lemma for deriving the proposed FMETS is introduced.

Lemma 1 (Finsler's Lemma [52]): For given matrices $P = P^T \in \mathbb{R}^{n\times n}$, and $B \in \mathbb{R}^{m\times n}$, the following conditions are equivalent:

$$(a)\ \ v^T P v < 0, \quad \text{for all}\ \ v \neq 0, Bv = 0;$$
$$(b)\ \ {B^{\perp}}^T P B^{\perp} < 0;$$
$$(c)\ \ \exists S \in \mathbb{R}^{n\times m}\ \ \text{such that}\ \ P + Sym(SB) < 0.$$

Notation 1: Throughout this article, a symmetric and positive (negative) definite matrix R is denoted by $R > 0$ (< 0). $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is used to show the set of all $n \times m$ real matrices. $\mathbb{Z}^+$ is the set of a positive integer. $*$ refers to the symmetric entries of a symmetry matrix. The subscripts T and $\perp$ represent the transpose and the null space of a matrix, respectively. Also, I_n and $0_{n\times m}$ are defined as a $n \times n$ identity matrix and a $n \times m$ zero matrix respectively.

B. A CLOSED-LOOP SYSTEM

In the FMETS, a closed-loop system that is combined with state feedback controller (3) in discrete-time system (2) is given as

$$x(k+1) = Ax(k) + BKx(k_s), \quad k \in [\,k_s, k_{s+1}\,). \tag{11}$$

By the property of the proposed event triggering condition (7), we can represent the triggering state $x(k_s)$ as the sum of delayed state and delayed triggering error as follows:

$$x(k_s) = x(k-i) + e(k-i), \quad i \in [\,0, N_s\,]. \tag{12}$$

Using (12), we can obtain a delayed triggering error $e(k-i)$ for $i \in [0, N_s]$ as follows:

$$\begin{aligned} x(k-i) + e(k-i) &= x(k-N_s) + e(k-N_s),\\ e(k-i) &= -x(k-i) + x(k-N_s) + e(k-N_s). \end{aligned} \tag{13}$$

Using (12), a closed-loop system (11) is also represented as

$$x(k+1) = Ax(k) + BKx(k-i) + BKe(k-i). \tag{14}$$

The equations (13) and (14) are used in Section IV-C.

C. THE STABILITY AND CO-DESIGN OF THE FMETS

This subsection provides the sufficient condition of the FMETS stability and co-design for a linear discrete-time systems.

Theorem 1 (Stability Analysis): For the given triggering threshold ρ and the forgetting level γ, the closed-loop system (11) is asymptotically stable under the proposed event triggering condition (7) if there exist symmetric matrices $P > 0$, $\Omega > 0$, and matrices $G_1, G_3, \cdots, G_{N+2}$ satisfying

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} \\ * & * & * & \Pi_{44} & \Pi_{45} \\ * & * & * & * & \Pi_{55} \end{bmatrix} < 0 \tag{15}$$

where submatrices of Π are given as

$$\Pi_{11} = P - G_1 - G_1^T$$
$$\Pi_{12} = G_1A + G_1BK$$
$$\Pi_{13} = 0_{n\times nN}$$
$$\Pi_{14} = G_1BK$$
$$\Pi_{15} = 0_{n\times nN}$$
$$\Pi_{22} = -P + \Omega_\rho$$
$$\Pi_{23} = \begin{bmatrix} G_3^T & \cdots & G_{N+2}^T \end{bmatrix}$$
$$\Pi_{24} = 0_{n\times n}$$
$$\Pi_{25} = 0_{n\times nN}$$
$$\Pi_{33} = \begin{bmatrix} \Xi_1 & 0_{n\times n} & \cdots & 0_{n\times n} \\ 0_{n\times n} & \Xi_2 & \cdots & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & \Xi_N \end{bmatrix} \in \mathbb{R}^{nN\times nN}$$
$$(\Xi_i \triangleq \gamma^i \Omega_\rho - G_{i+2} - G_{i+2}^T)$$
$$\Pi_{34} = \Pi_{23}^T$$
$$\Pi_{35} = \begin{bmatrix} -G_3 & 0_{n\times n} & \cdots & 0_{n\times n} \\ 0_{n\times n} & -G_4 & \cdots & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & -G_{N+2} \end{bmatrix} \in \mathbb{R}^{nN\times nN}$$
$$\Pi_{44} = -\Omega$$

$$\Pi_{45} = 0_{n\times nN}$$

$$\Pi_{55} = \begin{bmatrix} -\gamma\Omega & 0_{n\times n} & \cdots & 0_{n\times n} \\ 0_{n\times n} & -\gamma^2\Omega & \cdots & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & -\gamma^N\Omega \end{bmatrix} \in \mathbb{R}^{nN\times nN}.$$

Proof: To prove Theorem 1, we consider two cases, 1) $k_s \le k-N$ and 2) $k_s > k-N$. First, it is proved in case 1.

1) $k_s \le k-N$

When k_s is less than or equal to $k-N$, we can see that N_s is always N. Therefore, N is used instead of N_s for convenience. It is solved by choosing the Lyapunov functional as follows:

$$V(k) = x^T(k)Px(k). \tag{16}$$

Taking the time-derivative of (16), we calculate

$$\begin{aligned} \Delta V(k) &= x^T(k+1)Px(k+1) - x^T(k)Px(k) \qquad (17)\\ &= x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ &\quad + \sum_{i=0}^{N}\gamma^i e^T(k-i)\Omega e(k-i) \\ &\quad - \sum_{i=0}^{N}\gamma^i e^T(k-i)\Omega e(k-i) \\ &\le x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ &\quad + \sum_{i=0}^{N}\gamma^i x^T(k-i)\Omega_\rho x(k-i) \\ &\quad - \sum_{i=0}^{N}\gamma^i e^T(k-i)\Omega e(k-i). \qquad (18) \end{aligned}$$

The inequality of (18) can be represented as follows:

$$\Delta V(k) \le \xi_1^T(k)\Sigma_1\xi_1(k) \tag{19}$$

where Σ_1 and ξ_1 is given as

$$\Sigma_1 = diag\{P, -P+\Omega_\rho, \rho\Psi, -\Omega, -\Psi\}$$
$$\Psi = diag\{\gamma\Omega, \cdots, \gamma^N\Omega\}$$
$$\xi_1(k) = \begin{bmatrix} x(k+1) \\ x(k) \\ x(k-1) \\ \vdots \\ x(k-N) \\ e(k) \\ e(k-1) \\ \vdots \\ e(k-N) \end{bmatrix} \in \mathbb{R}^{(2N+3)n}.$$

To guarantee the asymptotic stability in a closed-loop system (14), (19) should be negative as follows:

$$\xi_1^T(k)\Sigma_1\xi_1(k) < 0. \tag{20}$$

Using the closed-loop system (14) and (13) yields

$$x(k+1) = Ax(k) + BKx(k-N) + BKe(k-N),$$
$$e(k-i) = -x(k-i) + x(k-N) + e(k-N).$$

Since $x(k+1)$ and $e(k-i)$ are represented as $x(k), x(k-1), \cdots, x(k-N)$ and $e(k-N)$, the following vector and matrix representation can be obtained:

$$\xi_2(k) = \Gamma_1\xi_1(k)$$
$$\xi_2(k) = \begin{bmatrix} x(k) \\ x(k-1) \\ \vdots \\ x(k-N) \\ e(k-N) \end{bmatrix} \in \mathbb{R}^{(N+2)n}$$
$$\Gamma_1 = \begin{bmatrix} A & 0_{n\times n} & \cdots & BK & BK \\ I_n & 0_{n\times n} & \cdots & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_n & \cdots & 0_{n\times n} & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & I_n & 0_{n\times n} \\ -I_n & 0_{n\times n} & \cdots & I_n & I_n \\ 0_{n\times n} & -I_n & \cdots & I_n & I_n \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & 0_{n\times n} & I_n \end{bmatrix} \in \mathbb{R}^{(2N+3)n\times(N+2)n}. \tag{21}$$

By using Lemma 1 and (21), the condition (20) is converted as follows:

$$\begin{aligned} &\xi_1^T(k)\Sigma_1\xi_1(k) = \xi_2^T(k)\Gamma_1^T\Sigma_1\Gamma_1\xi_2(k) < 0 \\ &\Gamma_1^T\Sigma_1\Gamma_1 < 0 \\ &\Sigma_1 + H_1\Gamma_1^\perp + \Gamma_1^{\perp T}H_1^T < 0 \qquad (22) \end{aligned}$$

where $\Gamma_1^\perp$ and H_1 are given as

$$\Gamma_1^{\perp T} = \begin{bmatrix} -I_n & 0_{n\times n} & \cdots & 0_{n\times n} \\ (A+BK)^T & I_n & \cdots & I_n \\ 0_{n\times n} & -I_n & \cdots & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & -I_n \\ (BK)^T & I_n & \cdots & I_n \\ 0_{n\times n} & -I_n & \cdots & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & -I_n \end{bmatrix} \in \mathbb{R}^{(2N+3)n\times(N+2)n}$$
$$H_1 = \begin{bmatrix} G_1 & 0_{n\times n} & \cdots & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & \cdots & 0_{n\times n} \\ 0_{n\times n} & G_3 & \cdots & 0_{n\times n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & G_{N+2} \end{bmatrix} \in \mathbb{R}^{(2N+3)n\times(N+2)n}.$$

Then, the inquality (22) is equivalent to (15).

2) $k_s > k-N$

To prove Theorem 1 in $k_s > k-N$, we compare two cases of the memory size, N and $N-1$. The sufficient LMI conditions (15) in $N-1$ and N are defined as $\Pi_{(N-1)}$ and

$\Pi_{(N)}$, respectively. Then, the relationship between $\Pi_{(N-1)}$ and $\Pi_{(N)}$ is obtained as

$$\begin{aligned}
\Pi_{(N),11} &= \Pi_{(N-1),11}\\
\Pi_{(N),12} &= \Pi_{(N-1),12}\\
\Pi_{(N),13} &= \begin{bmatrix}\Pi_{(N-1),13} & 0_{n\times n}\end{bmatrix}\\
\Pi_{(N),14} &= \Pi_{(N-1),14}\\
\Pi_{(N),15} &= \begin{bmatrix}\Pi_{(N-1),15} & 0_{n\times n}\end{bmatrix}\\
\Pi_{(N),22} &= \Pi_{(N-1),22}\\
\Pi_{(N),23} &= \begin{bmatrix}\Pi_{(N-1),23} & 0_{n\times n}\end{bmatrix}\\
\Pi_{(N),24} &= \Pi_{(N-1),24}\\
\Pi_{(N),25} &= \begin{bmatrix}\Pi_{(N-1),25} & 0_{n\times n}\end{bmatrix}\\
\Pi_{(N),33} &= \begin{bmatrix}\Pi_{(N-1),33} & 0_{(N-1)n\times n}\\ & \gamma^N\Omega_\rho - G_{N+2} - G_{N+2}^T\end{bmatrix}\\
\Pi_{(N),34} &= \begin{bmatrix}\Pi_{(N-1),34}\\ G_{N+2}\end{bmatrix}\\
\Pi_{(N),35} &= \begin{bmatrix}\Pi_{(N-1),35} & 0_{(N-1)n\times n}\\ * & -G_{N+2}\end{bmatrix}\\
\Pi_{(N),44} &= \Pi_{(N-1),44}\\
\Pi_{(N),45} &= \begin{bmatrix}\Pi_{(N-1),45} & 0_{n\times n}\end{bmatrix}\\
\Pi_{(N),55} &= \begin{bmatrix}\Pi_{(N-1),55} & 0_{(N-1)n\times n}\\ * & -\gamma^N\Omega\end{bmatrix}.
\end{aligned}$$

It means that $\Pi_{(N)}$ contains every block matrices of $\Pi_{(N-1)}$. By using the property of a negative definite matrix, we show that $\Pi_{(N-1)}$ is also a negative definite matrix if $\Pi_{(N)}$ is a negative definite matrix. First, by the definition of a negative definite matrix, $\Pi_{(N)}$ is represented as follows:

$$\zeta^T\Pi_{(N)}\zeta < 0, \quad \text{for } \zeta \neq 0.$$

Also, we choose ζ as follows:

$$\begin{aligned}
\zeta &= \begin{bmatrix}\zeta_1\\ 0_{n\times 1}\\ \zeta_2\\ 0_{n\times 1}\end{bmatrix} \in \mathbb{R}^{(2N+3)n}\\
\zeta_1 &\neq 0 \in \mathbb{R}^{(N+1)n}\\
\zeta_2 &\neq 0 \in \mathbb{R}^{Nn}.
\end{aligned}$$

Then, we can show that $\Pi_{(N-1)}$ is a negative definite matrix as follows:

$$\begin{aligned}
\zeta^T\Pi_{(N)}\zeta &= \begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix}^T \Pi_{(N-1)} \begin{bmatrix}\zeta_1\\ \zeta_2\end{bmatrix} < 0\\
\Pi_{(N-1)} &< 0
\end{aligned}$$

Therefore, by the chain rule, if $\Pi_{(N)}$ is negative definite, $\Pi_{(i)}$ is also negative definite for all $i \in \{0, \cdots, N\}$ and it means that the proposed event triggering condition (7) in $k - k_s < N$ guarantees the asymptotic stability in a closed-loop system (11). □

Theorem 2 (Controller Design): For given scalars b_1, b_2 and tuning parameters ρ and γ, the closed-loop system (11) is asymptotically stable under the proposed event triggering condition (7) if there exist symmetric matrices $P > 0$, $\Omega > 0$, and matrices M, W, $G_1, G_3, \cdots, G_{N+2}$ satisfying

$$\Phi = \begin{bmatrix}\Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16}\\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26}\\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36}\\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46}\\ * & * & * & * & \Phi_{55} & \Phi_{56}\\ * & * & * & * & * & \Phi_{66}\end{bmatrix} < 0 \quad (23)$$

where submatrices of Φ are given as

$$\begin{aligned}
\Phi_{11} &= P - G_1 - G_1^T\\
\Phi_{12} &= G_1A + b_1BM\\
\Phi_{13} &= 0_{n\times nN}\\
\Phi_{14} &= b_1BM\\
\Phi_{15} &= 0_{n\times nN}\\
\Phi_{16} &= -b_1BW + G_1B\\
\Phi_{22} &= -P + \Omega_\rho\\
\Phi_{23} &= \begin{bmatrix}G_3^T & \cdots & G_{N+2}^T\end{bmatrix}\\
\Phi_{24} &= 0_{n\times n}\\
\Phi_{25} &= 0_{n\times nN}\\
\Phi_{26} &= b_2M^T\\
\Phi_{33} &= \begin{bmatrix}\Xi_1 & 0_{n\times n} & \cdots & 0_{n\times n}\\ 0_{n\times n} & \Xi_2 & \cdots & 0_{n\times n}\\ \vdots & \vdots & \ddots & \vdots\\ 0_{n\times n} & 0_{n\times n} & \cdots & \Xi_N\end{bmatrix} \in \mathbb{R}^{nN\times nN}\\
&\quad (\Xi_i \triangleq \gamma^i\Omega_\rho - G_{i+2} - G_{i+2}^T)\\
\Phi_{34} &= \Phi_{23}^T\\
\Phi_{35} &= \begin{bmatrix}-G_3 & 0_{n\times n} & \cdots & 0_{n\times n}\\ 0_{n\times n} & -G_4 & \cdots & 0_{n\times n}\\ \vdots & \vdots & \ddots & \vdots\\ 0_{n\times n} & 0_{n\times n} & \cdots & -G_{N+2}\end{bmatrix} \in \mathbb{R}^{nN\times nN}\\
\Phi_{36} &= 0_{nN\times m}\\
\Phi_{44} &= -\Omega\\
\Phi_{45} &= 0_{n\times nN}\\
\Phi_{46} &= b_2M^T\\
\Phi_{55} &= \begin{bmatrix}-\gamma\Omega & 0_{n\times n} & \cdots & 0_{n\times n}\\ 0_{n\times n} & -\gamma^2\Omega & \cdots & 0_{n\times n}\\ \vdots & \vdots & \ddots & \vdots\\ 0_{n\times n} & 0_{n\times n} & \cdots & -\gamma^N\Omega\end{bmatrix} \in \mathbb{R}^{nN\times nN}\\
\Phi_{56} &= 0_{nN\times m}\\
\Phi_{66} &= -b_2W - b_2W^T.
\end{aligned}$$

Then, the controller gain is solved by $K = W^{-1}M$.

Proof: we can represent the inequality (15) in Theorem 1 as follows:

$$\begin{aligned}
\Pi &= \Sigma_1 + H_1\Gamma_1^\perp + \Gamma_1^{\perp T}H_1^T = \Gamma_2^T\Sigma_2\Gamma_2 < 0\\
\Sigma_2 &= \begin{bmatrix}\Sigma_1 + H_1\Gamma_1^\perp + \Gamma_1^{\perp T}H_1^T & 0_{(2N+3)n\times m}\\ * & 0_{m\times m}\end{bmatrix}
\end{aligned}$$

$$\Gamma_2 = \begin{bmatrix} I_{n\times n} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{n\times n} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{nN\times nN} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{n\times n} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{nN\times nN} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{n\times n} \\ 0_{m\times n} & K & 0_{m\times nN} & K & 0_{m\times nN} & 0_{m\times n} \end{bmatrix}. \quad (24)$$

By using Lemma 1, the equivalent inequality is solved as follows:

$$\begin{aligned} &\Sigma_2 + H_2\Gamma_2^{\perp} + \Gamma_2^{\perp T} H_2^T < 0 \\ &\Gamma_2^{\perp} = \begin{bmatrix} 0_{m\times n} & K & 0_{m\times nN} & K & 0_{m\times nN} & -I_{m\times m} \end{bmatrix} \\ &H_2 = \begin{bmatrix} b_1BW - G_1B \\ 0_{n\times m} \\ 0_{nN\times m} \\ 0_{n\times m} \\ 0_{nN\times m} \\ b_2W \end{bmatrix} \end{aligned} \quad (25)$$

Then, WK is replaced as M. *i.e.*, $WK \triangleq M$. Therefore, inequality (25) is equal to (23). □

Remark 2: Using Theorem 2, it is possible to design the state feedback controller and the finite memory event triggering condition. By adjusting tuning parameters ρ, γ, and N at the same time, it is easy to adjust the convergence rate and the number of the triggering. Also, the FMETS is the generalized framework that includes the conventional RETS and the IETS because the RETS and the IETS are reduced to the proposed FMETS when N is set to be zero and ∞, respectively. Therefore, the FMETS provides the additional design freedom to adjust the convergence rate and the number of the triggering compared with the conventional RETS and the IETS. The characteristics of each parameter are verified by the following section.

V. NUMERICAL EXAMPLES

In this section, three numerical examples are introduced to describe the property and the effectiveness of the FMETS.

Example 1: In example 1, the effectiveness of the proposed event triggering condition is analyzed. Consider the linear discrete system (2) with following model parameters and the initial state $x_0^T = \begin{bmatrix} 5 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 1.1 & 0 \\ 1 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}.$$

The weighting matrix Ω and the gain matrix K are solved with $\rho = 0.3$, $\gamma = 1$ and $b_1 = b_2 = 1$ in case of $N = 4$, as follows:

$$\Omega = \begin{bmatrix} 0.7124 & -0.1621 \\ -0.1621 & 0.4448 \end{bmatrix}, \quad K = \begin{bmatrix} -0.1534 & -0.1461 \end{bmatrix}.$$

To show the effectiveness of the proposed event triggering condition, three simulations are performed with all the same conditions except that N is 0, 1 and 4, respectively. $N = 0$ means that only current states are used in the event triggering condition, or the FMETS is reduced to the RETS. Relevant simulation results are shown together with state trajectories in Figures 4 and 5. The upper parts of Figures 4(a), (b), and (c) show the state trajectories in two-dimensional state space as Section III-C, and the lower parts do the state trajectories over time. It is observed that the states of the system reach the zero and the asymptotic stability is proved. In Figure 4(a), the triggering region of the RETS is represented as blue ellipsoids, or the triggering occurs if the states escape this region. In Figures 4(b) and 4(c), the triggering region of the FMETS is drawn with red ellipsoids, which is larger than that of Figure 4(a). Also, as N increases, the size of the triggering region becomes large. Figure 5 shows the control input and inter-event times. Additionally, the triggering time points are also observed. The control input is kept until the triggering occurs and is updated when the triggering occurs. In Table 1, N_t represents the number of event triggering times. As the memory size increases, the number of event triggering times decreases. It can be said that the proposed condition can suppress unnecessary even triggering as much as possible while guaranteeing the system stability.

TABLE 1. The number of the triggering N_t in each N.

N	0	1	4
N_t	61	47	36

Example 2: In this example, we compare the proposed FMETS with the IETS to illustrate the effectiveness of the proposed FMETS. Also, we analyze the characteristics of parameters used in the proposed event triggering condition and suggest the appropriate design methods. Consider a discretized model (2) with the following system matrices and the initial states $x_0^T = \begin{bmatrix} 15 & -3 & -7 \end{bmatrix}$

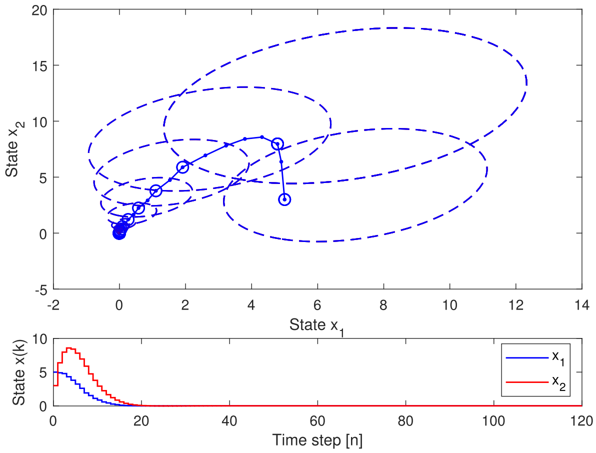
$$A = \begin{bmatrix} 0.85 & 0 & 0.1 \\ 0.01 & 0.96 & 0 \\ 0 & 0 & 1.0 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0 & 0 \\ -0.2 & -0.1 & 0 \\ -0.1 & 0.1 & 0.1 \end{bmatrix}.$$

In a discrete-time system, the event triggering condition of the IETS is described as

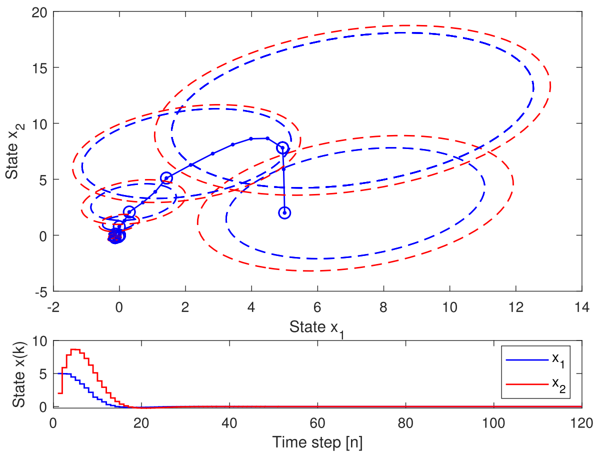
$$\sum_{i=k_s}^{k} |e(i)|^2 \leq \sum_{i=k_s}^{k} \rho |x(i)|^2. \quad (26)$$

The event triggering condition of the IETS is the same as that of the proposed FMETS in case that $N = \infty$, $\gamma = 1$ and $\Omega = I_n$. To verify the effectiveness of the FMETS, it is designed to have the same number of the triggering as the IETS *i.e.*, $N_t = 10$. The triggering threshold ρ is chosen to 0.2, N is 2 and γ is 1.4. The weighting matrix Ω is set to I_3. The control gains K_F and K_I of the FMETS and IETS are given as

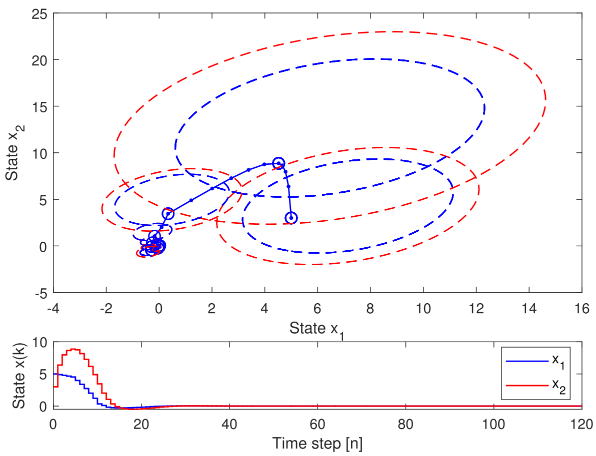
$$K_F = \begin{bmatrix} 0.0185 & 0.0249 & 0.0256 \\ -0.0241 & 0.0194 & -0.0196 \\ 0.0070 & -0.0091 & -0.0451 \end{bmatrix},$$



(a) States trajectories for $N = 0$.



(b) States trajectories for $N = 1$.



(c) States trajectories for $N = 4$.

FIGURE 4. States trajectories in Example 1.

$$K_I = \begin{bmatrix} 0.0249 & 0.0472 & 0.0456 \\ -0.0373 & 0.0294 & -0.0428 \\ 0.0140 & -0.0107 & -0.0772 \end{bmatrix}.$$

The simulation result is given in Figure 6. The triggering events of the IETS and the FMETS are distributed differently to each other in the transient response, but their inter-event times are similar to each other in the steady-state response. While the IETS requires an infinite amount of memory for the integral calculation, the FMETS can use a finite memory to obtain the similar number of triggering times. In other words, the FMETS has an advantage compared with the IETS in terms of the memory cost.

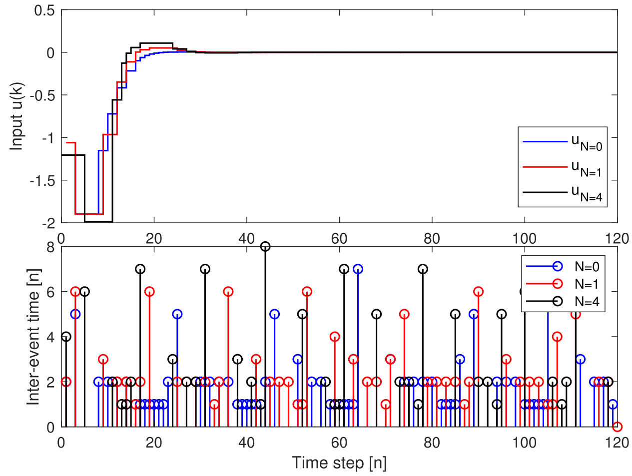


FIGURE 5. The control input and inter-event time in Example 1.

Second, we analyze the characteristics of tuning parameters used in the proposed event triggering condition. For analysis, three FEMTSs are constructed and compared with each other. Parameters and the number of the triggering in three FMETSs are described Table 2 and the weighting matrix Ω and the control gain K are given as

$$\Omega_A = \begin{bmatrix} 0.2462 & 0.0137 & -0.1579 \\ 0.0137 & 0.1405 & 0.0010 \\ -0.1579 & 0.0010 & 0.1562 \end{bmatrix},$$

$$\Omega_B = \begin{bmatrix} 0.2873 & 0.0305 & -0.1659 \\ 0.0305 & 0.1810 & -0.0059 \\ -0.1659 & -0.0059 & 0.1678 \end{bmatrix},$$

$$\Omega_C = \begin{bmatrix} 0.1433 & 0.0145 & -0.0802 \\ 0.0145 & 0.0894 & -0.0011 \\ -0.0802 & -0.0011 & 0.0809 \end{bmatrix},$$

$$K_A = \begin{bmatrix} 0.1031 & 0.2419 & 0.0869 \\ -0.0115 & 0.0635 & -0.1230 \\ 0.1530 & -0.0213 & -0.2748 \end{bmatrix},$$

$$K_B = \begin{bmatrix} 0.0197 & 0.1013 & 0.0721 \\ -0.0141 & -0.0081 & -0.0668 \\ 0.0583 & 0.0082 & -0.1311 \end{bmatrix},$$

$$K_C = \begin{bmatrix} 0.0279 & 0.0897 & 0.0516 \\ -0.0105 & 0.0056 & -0.0574 \\ 0.0540 & 0.0075 & -0.1043 \end{bmatrix}.$$

States trajectories and the inter-event time are shown in Figure 7. The simulation A, B and C in Table 2 represent the effect of the triggering threshold ρ, the forgetting level γ and the memory size N, respectively. All the simulations have a similar number of the triggering to verify the characteristics

TABLE 2. **The parameter and the number of the triggering in three FMETSs.**

	N	ρ	γ	N_t
Simul A	0	0.95	1	15
Simul B	1	0.15	4	15
Simul C	4	0.15	1	14

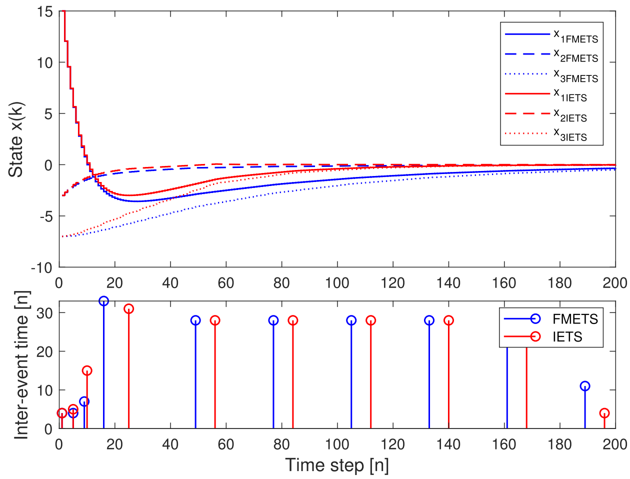


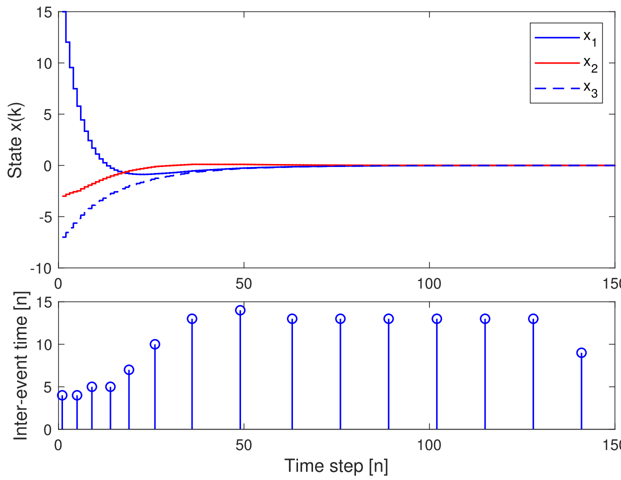
FIGURE 6. **States trajectories and the inter-event time in the FMETS and the IETS.**

TABLE 3. **The network resource adjustability.**

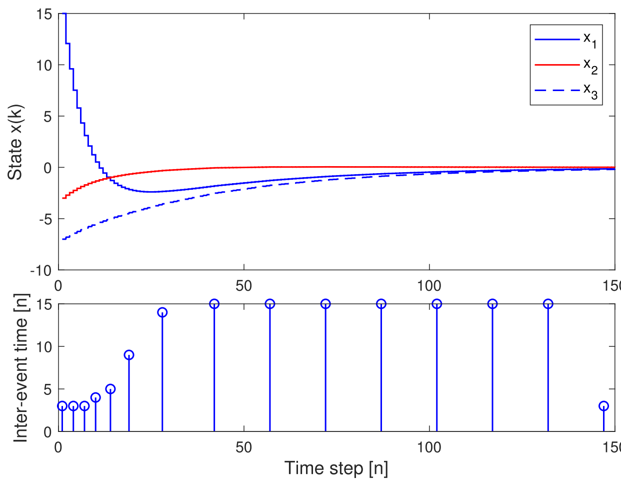
	ρ	γ	N
Transient region	strong	weak	weak
Steady-state region	weak	strong	strong

of each parameter. In the simulation A, the triggering occurs less in the transient response and more in the steady-state response compared with B and C. On the other hand, the triggering occurs more in the transient response and less in the steady-state response in case of the simulation B and C. In (10), ρ and $x(k_s)$ determine the size of the triggering region. In the transient response, the latest triggering state $x(k_s)$ has a relatively large value and ρ changes the triggering region effectively. Conversely, if $x(k_s)$ is close to 0, the effect of ρ decreases. Instead, γ and N have play the role of additional margins, so they are effective in a steady-state response.

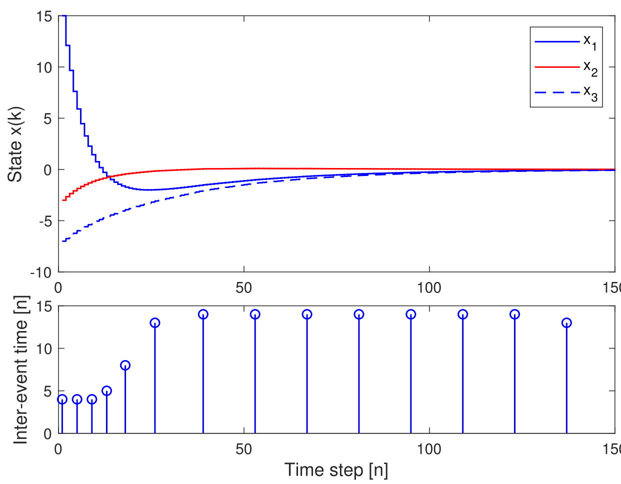
Remark 3: In the FMETS, there are three ways to reduce the number of the triggering. 1) increase the ρ value, 2) increase the γ value and 3) increase the N value. In (4) and (7), the weighted norms of states play the role of the upper bound. ρ determines how much the weighted norm of the state $|\Omega^{\frac{1}{2}}x(k)|^2$ is proportional to the upper bound of the event triggering conditions. Therefore, if $|\Omega^{\frac{1}{2}}x(k)|^2$ is large, the number of the triggering is sensitive to ρ As a result, ρ is effective to adjust the network resources in the transient region where $|\Omega^{\frac{1}{2}}x(k)|^2$ is large relatively. On the other hand, γ and N adjusts the amount of event triggering residuals ε_k. Therefore, γ and N are effective to adjust the



(a) States trajectories and the inter-event time in simul A.



(b) States trajectories and the inter-event time in simul B.



(c) States trajectories and the inter-event time in simul C.

FIGURE 7. **States trajectories and the inter-event time in Example 2.**

network resources in the steady-state region where $|\Omega^{\frac{1}{2}} x(k)|^2$ is small relatively. Table 3 represents the network resource adjustability of ρ, γ and N.

Example 3: In this example, we extend the proposed FMETS to a continuous-time system with the sampler to avoid the Zeno effect. System matrices of a continuous-time system (1) are given as

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -18.18 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -17.86 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 & 0 \\ 0 & 515.38 \\ 0 & 0 \\ 0 & 517.07 \end{bmatrix}$$

where the initial state $x_0^T = \begin{bmatrix} 1.0 & 0.7 & 0.3 & 0.1 \end{bmatrix}$ and the sampling time $h = 0.1$. By employing the sampler, we can obtain a discretized model (2) with following system matrices [12]:

$$A = \begin{bmatrix} 1 & 0.046 & 0 & 0 \\ 0 & 0.162 & 0 & 0 \\ 0 & 0 & 1 & 0.047 \\ 0 & 0 & 0 & 0.168 \end{bmatrix}, \quad B = \begin{bmatrix} 0.486 & 0 \\ 16.926 & 0 \\ 0 & 0.490 \\ 0 & 17.098 \end{bmatrix}.$$

The weighting matrix Ω and the gain matrix K are solved with $\rho = 0.1, \gamma = 1, N = 2$ and $b_1 = b_2 = 1$.

$$\Omega = \begin{bmatrix} 0.016 & 0.024 & 0 & 0 \\ 0.024 & 1.066 & 0 & 0 \\ 0 & 0 & 0.016 & 0.024 \\ 0 & 0 & 0.024 & 1.067 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.005 & -0.006 & 0 & 0 \\ 0 & 0 & -0.005 & -0.006 \end{bmatrix}.$$

States trajectories and the time interval are shown in Figure 8. It can be seen that the continuous-time system model converges to 0 asymptotically using K and Ω designed in the discretized model, and the Zeno effect does not occur by guaranteeing the minimum inter-event time.

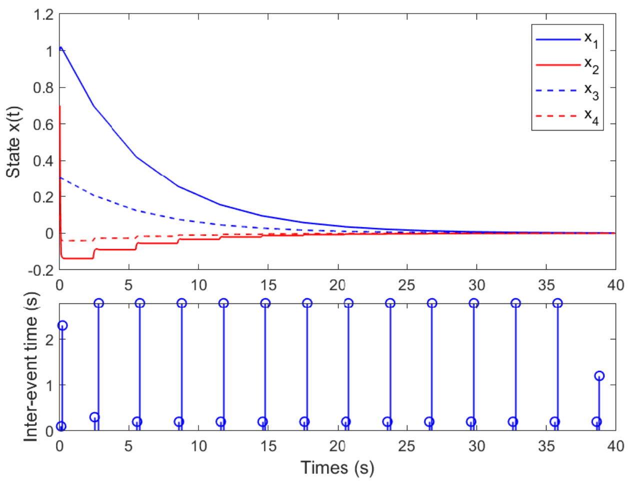


FIGURE 8. States trajectories and the inter-event times of a continuous-time networked system.

VI. CONCLUSION

This article proposed a more configurable event triggering control scheme that employs a finite memory for a networked control system. The proposed FMETS guarantees the closed-loop stability using the current and past states on the recent finite time horizon. Three tuning design parameters are provided to more effectively and flexibly balance the trade-off between the performance and the network resources in transient and steady-state responses. In addition, the FMETS can avoid the undesirable Zeno effect automatically in a continuous-time system due to a sample-and-hold approach. Three numerical examples were employed to verify the effectiveness of the FMETS. It is believed that the proposed method could be a good alternative to maintaining good performance while making the most of communication resources. Future work will be devoted to consider the time-delay and the packet loss in NCSs. In addition, we will find the optimal tuning parameters in the limited network resources and the convergence rate.

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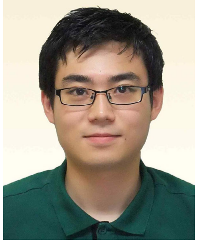

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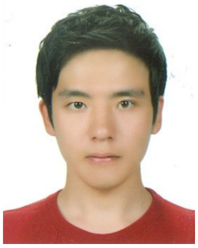

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