손실 모드 공명 광섬유 굴절률 센서의 특성 분석을 위한 이론적 연구

Theoretical study on lossy mode resonance fiber-optic refractometer

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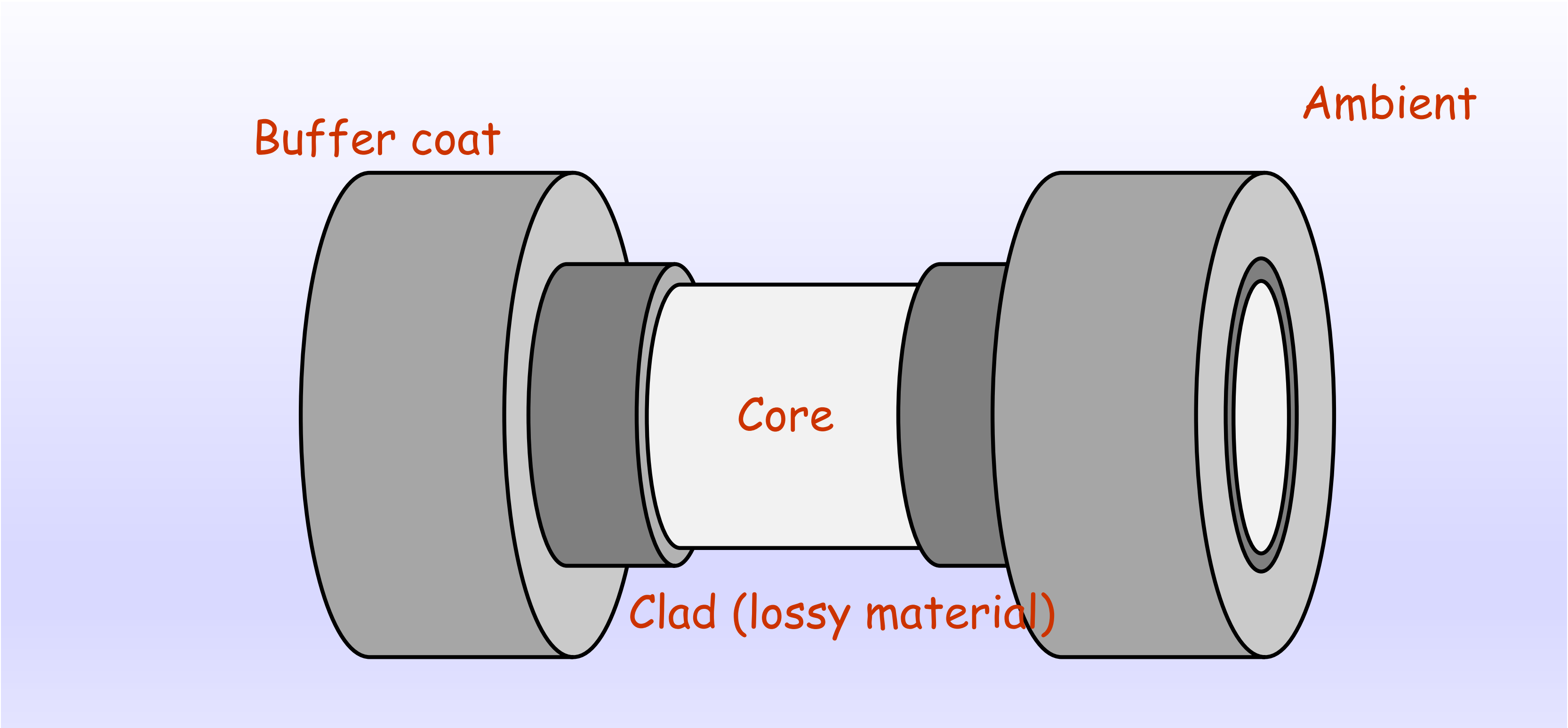
Requirements for an effective fiber-optic biosensor

- Sensitive to changes in refractive index (or thickness) of the ambient (sample)
- Dynamic range of the sensor can be tuned for each channel
- Signal highly dependent on incident wavelength, but not on the angle of internal reflection (LMR[1] vs SPR[2])
- Routine surface chemistry (e.g. silanization) must be applicable

[1]LMR: Lossy-mode resonance

[2]SPR: Surface plasmon resonance

Lossy mode resonance fiber-optic sensor layout

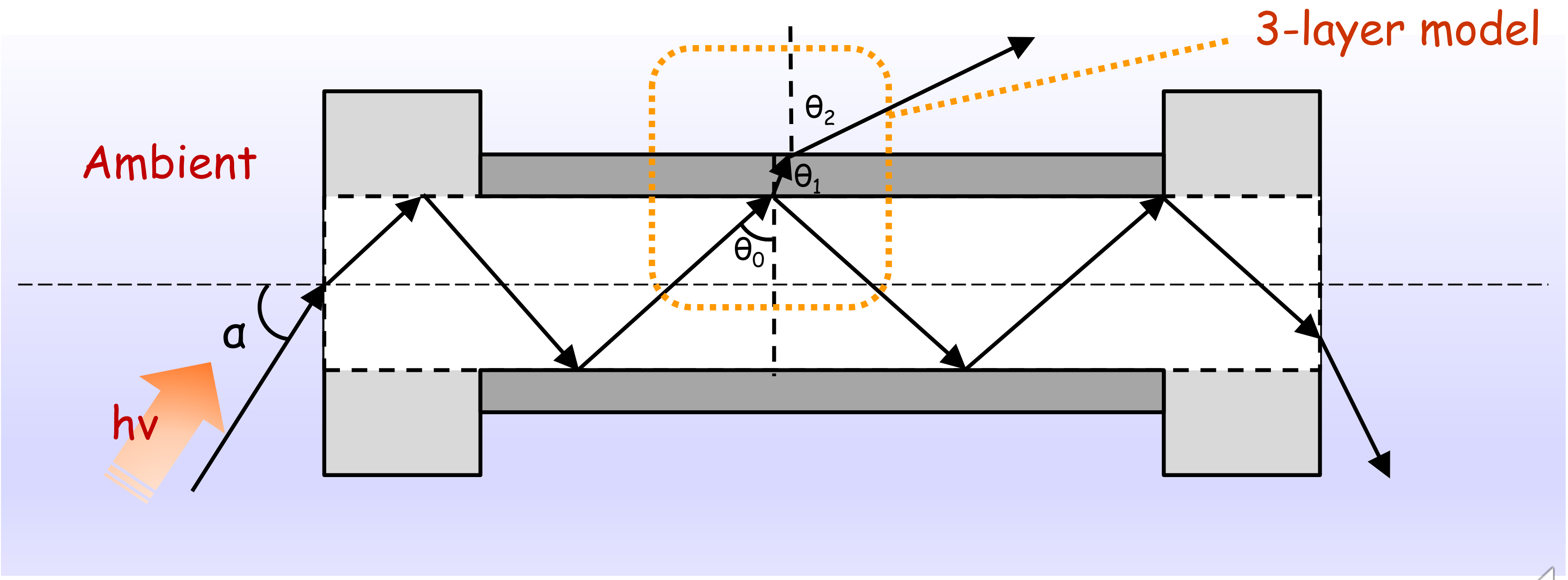


1. Core: light-guiding layer (SiO_2)
2. Clad: sensing layer (lossy material, e.g., TCOs)
3. Ambient: sample solution (buffer solution, blood plasma, etc.)

3-layer model (core-clad-ambient)

Optical properties of each layers are characterized completely by

1. Intrinsic constants: $\varepsilon_j(\lambda)$, $\mu_j(\lambda)$, and d_j
2. External parameters: θ_j and λ

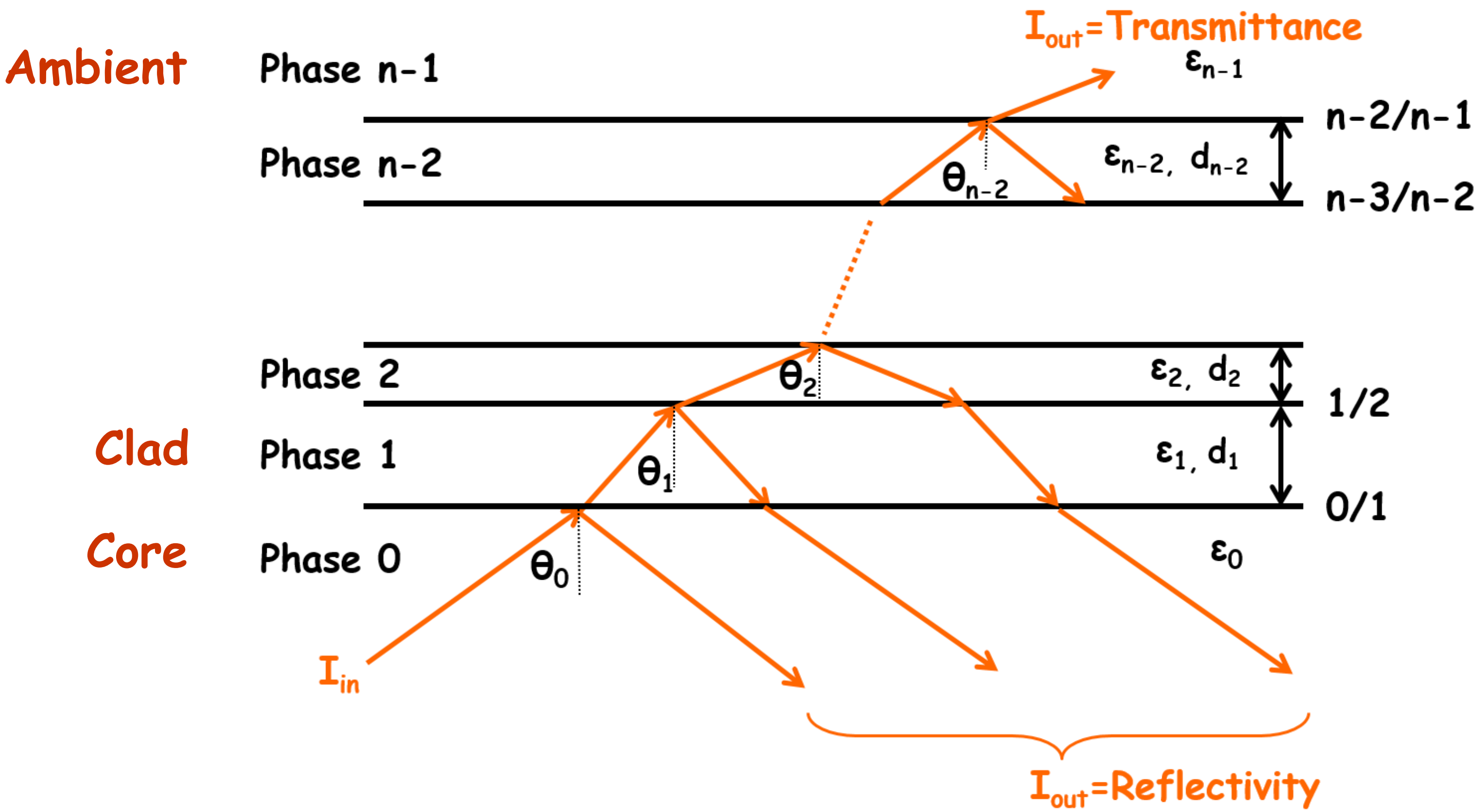


Fresnel equations for n-stratified layers

(Complex) dielectric function ε_j is defined as

$$\epsilon_j = N_j^2 = (n_j + ik_j)^2$$

where n_j: refractive index, k_j: extinction coefficient with layer thickness d_j



Electromagnetic fields in a stratified medium

(W. N. Hansen, JOSA 58(3) 380-390 (1968))

Tangential components of the fields (u, v) at the first and last boundaries are related with the characteristic matrix M

$$\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = M \begin{pmatrix} u_{n-2} \\ v_{n-2} \end{pmatrix} \qquad M = \prod_{j=1}^{n-2} M_j = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

The characteristic matrix for jth layer is given by

$$M_j = \begin{pmatrix} \cos\beta_j & \frac{-i}{p_j}\sin\beta_j \\ -ip_j\sin\beta_j & \cos\beta_j \end{pmatrix} \quad \text{for} \quad j = 1, \ldots, n-2$$

where

$$\begin{aligned} p_j &= \sqrt{\frac{\epsilon_j}{\mu_j}}\cos\theta_j \quad \text{(TE-mode)} \\ &= \sqrt{\frac{\mu_j}{\epsilon_j}}\cos\theta_j \quad \text{(TM-mode)} \end{aligned}$$

and

$$\beta_j = \left(\frac{2\pi}{\lambda}\right)\xi_j d_j$$

with $\xi_j = (N_j^2 - N_0^2 \sin^2\theta_0)^{1/2} = N_j \cos\theta_j$ **(Snell's law)**

Reflectance and transmittance

For non-magnetic materials (μ_j=1),

$$p_j = \sqrt{\epsilon_j}\cos\theta_j = \sqrt{N_j^2 - N_0^2 \sin\,\theta_0^2} \quad \textbf{(TE-mode)}$$

$$= \frac{\cos\theta_j}{\sqrt{\epsilon_j}} = \sqrt{1 - \frac{N_0^2}{N_j^2}\sin\,\theta_0^2} \quad \textbf{(TM-mode)}$$

The reflection- and transmission coefficients are

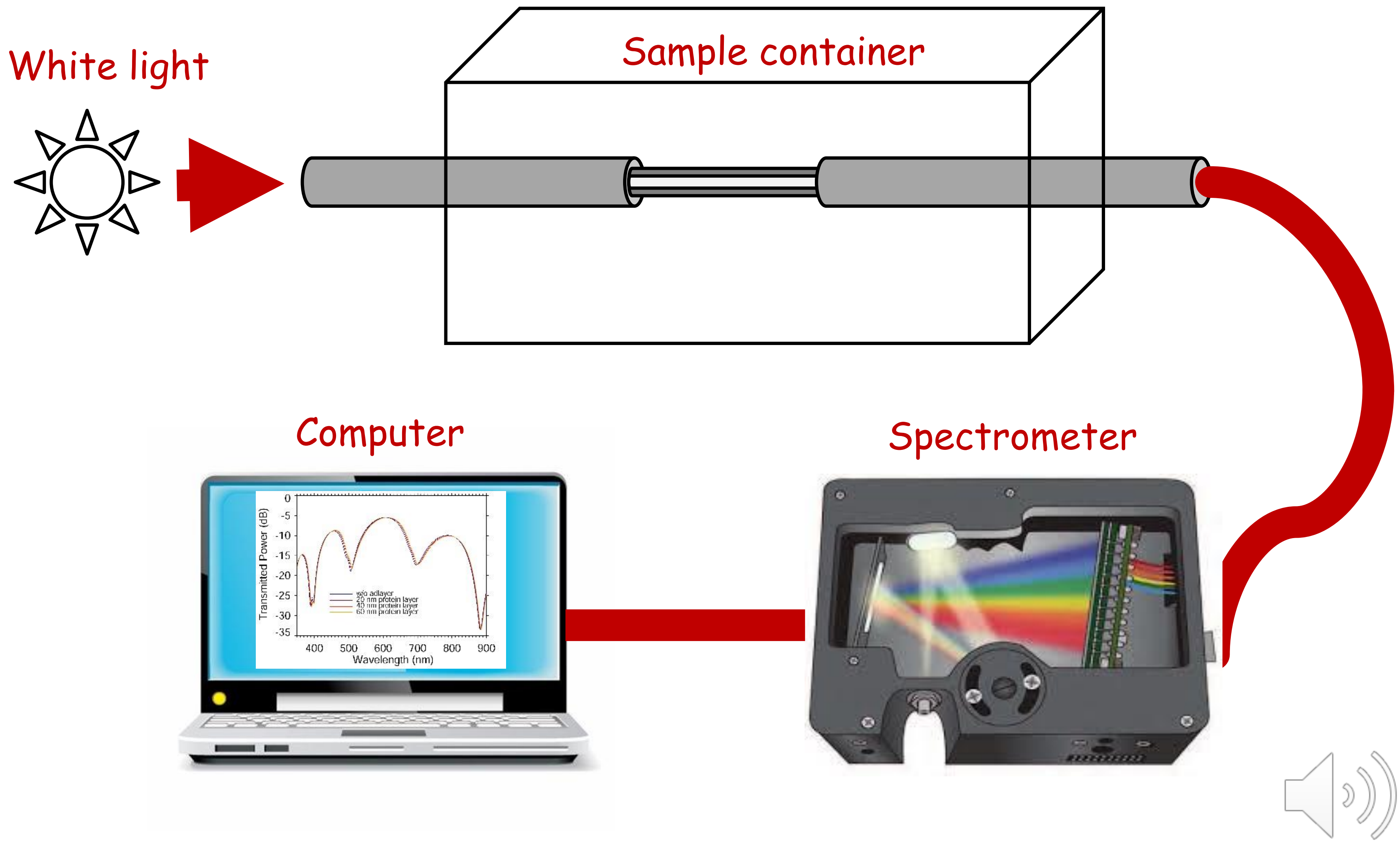
$$r = \frac{(m_{11} + m_{12}p_{n-1})p_0 - (m_{21} + m_{22}p_{n-1})}{(m_{11} + m_{12}p_{n-1})p_0 + (m_{21} + m_{22}p_{n-1})}$$

$$t = \frac{2p_0}{(m_{11} + m_{12}p_{n-1})p_0 + (m_{21} + m_{22}p_{n-1})}$$

And the reflectance and transmittance are

$$R(\theta_0, \lambda) = rr^* \qquad T(\theta_0, \lambda) = tt^*$$

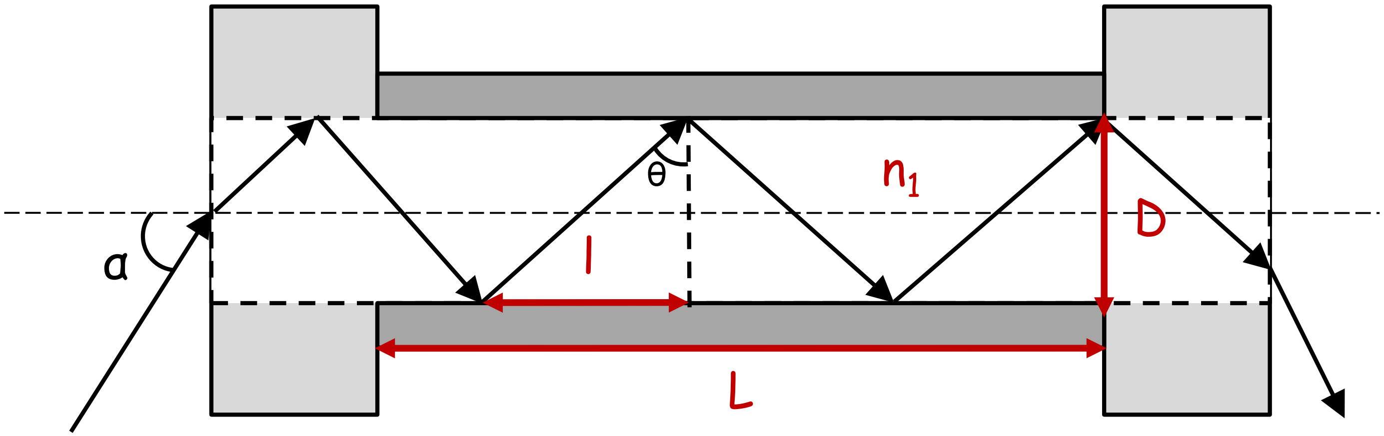
Schematic illustration of transmission measurements (not scaled)



Power distribution

The number of ray reflections in a sensing region is

$$N(\theta) = \frac{L}{D\ \tan\theta}$$



and the power distribution is (for the core RI n_1)

$$dP \propto \frac{n_1^2 \sin\theta \cos\theta}{(1 - n_1^2 \cos^2\theta)^2} d\theta$$

Reflectance and transmitted power

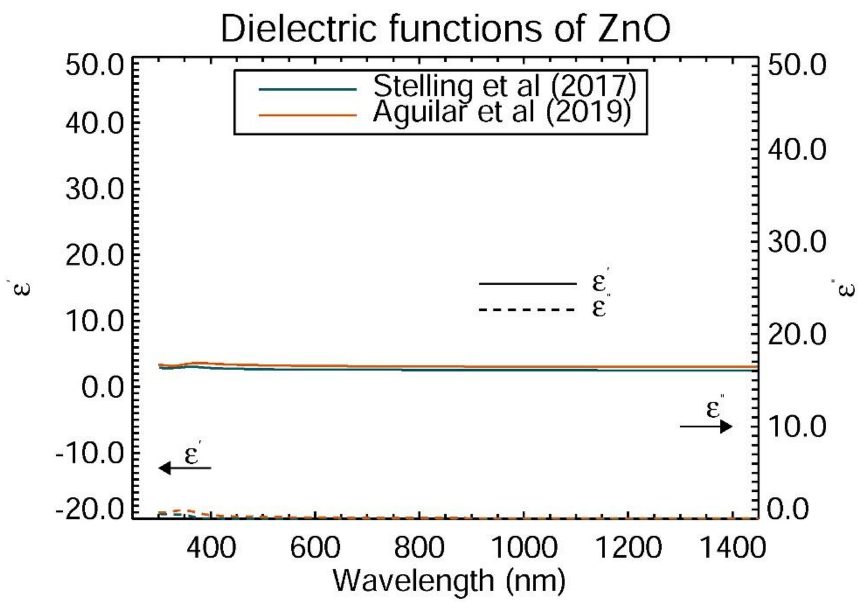
The normalized transmitted power of all guided rays is

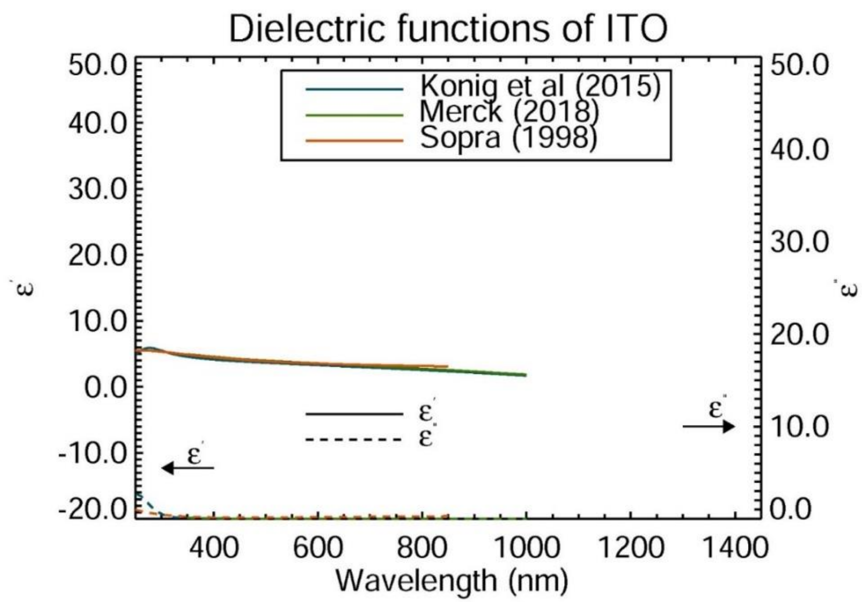
$$P = \frac{\int_{\theta_{cr}}^{\frac{\pi}{2}} R^{N(\theta)} p(\theta) d\theta}{\int_{\theta_{cr}}^{\frac{\pi}{2}} p(\theta) d\theta} \quad \text{where} \quad p(\theta) = \frac{n_1^2 \sin\theta \cos\theta}{(1 - n_1^2 \cos^2\theta)^2}$$

with $R(\theta,\lambda)$, $N(\theta)$, and the critical angle $\theta_{cr.}$

- Calculation conditions:
  1. All materials involved are homogeneous and isotropic
  2. Core material: SiO_2, Ø=400 μm multi-mode fiber (MMF)
  3. Clad : ITO (lossy material), thickness=440 nm
  4. Length of the sensing region=40 mm
  5. Refractive index of the proteins=1.41

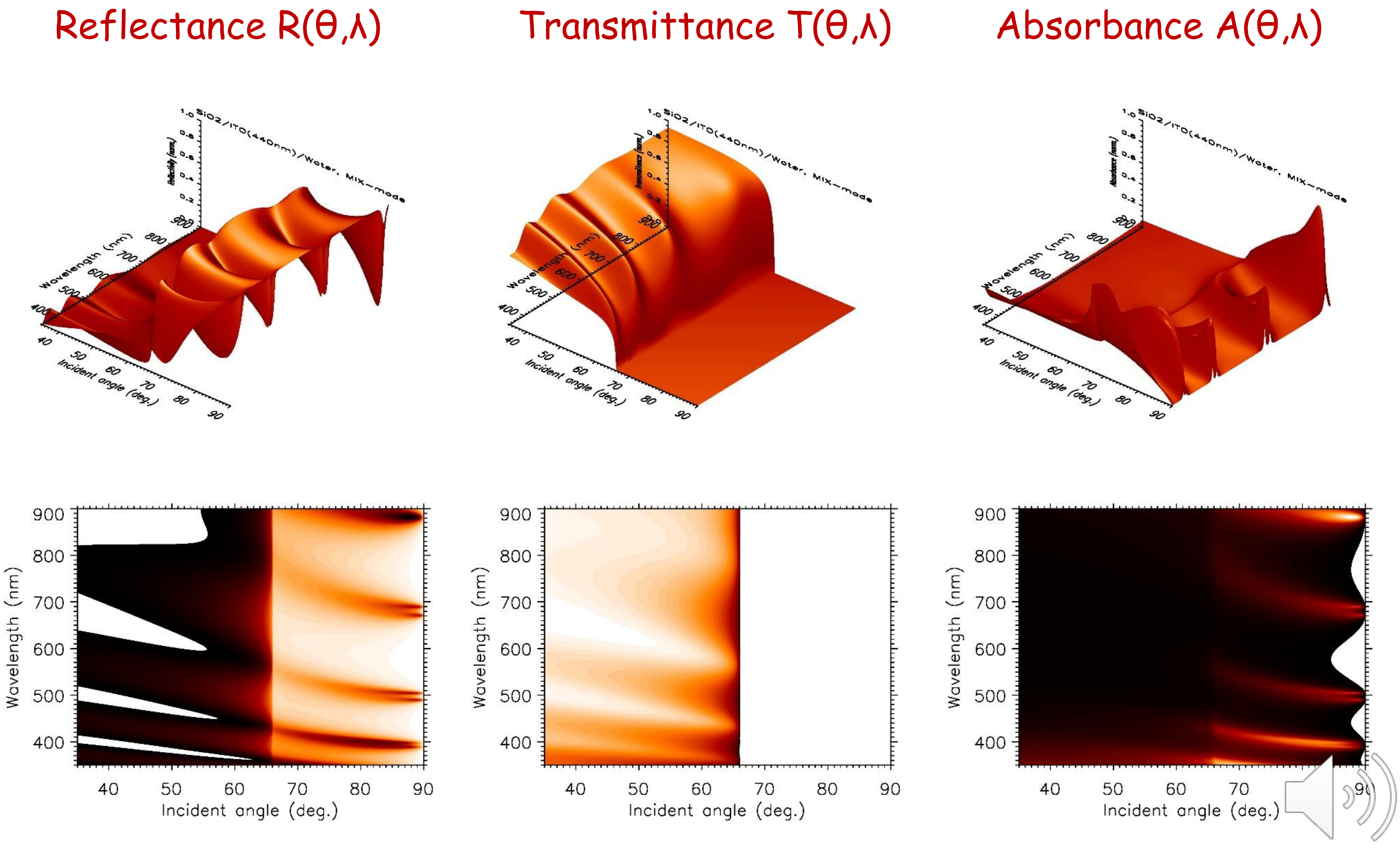
Dielectric function ε(λ) of TCOs





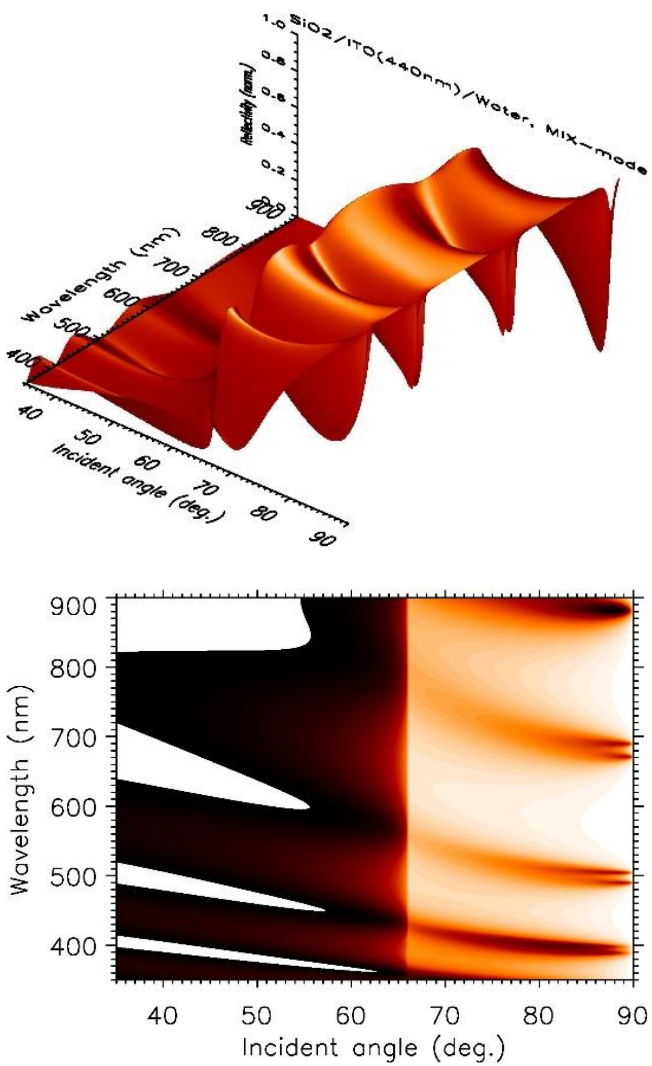
Because dielectric functions ε(λ) of the transparent conductive oxides (TCOs) strongly depend on the method of deposition and any impurities, the measured data were interpolated and used only for further calculations

Reflectance, transmittance and absorbance for SiO2/ITO(440nm)/water (MIX-mode)

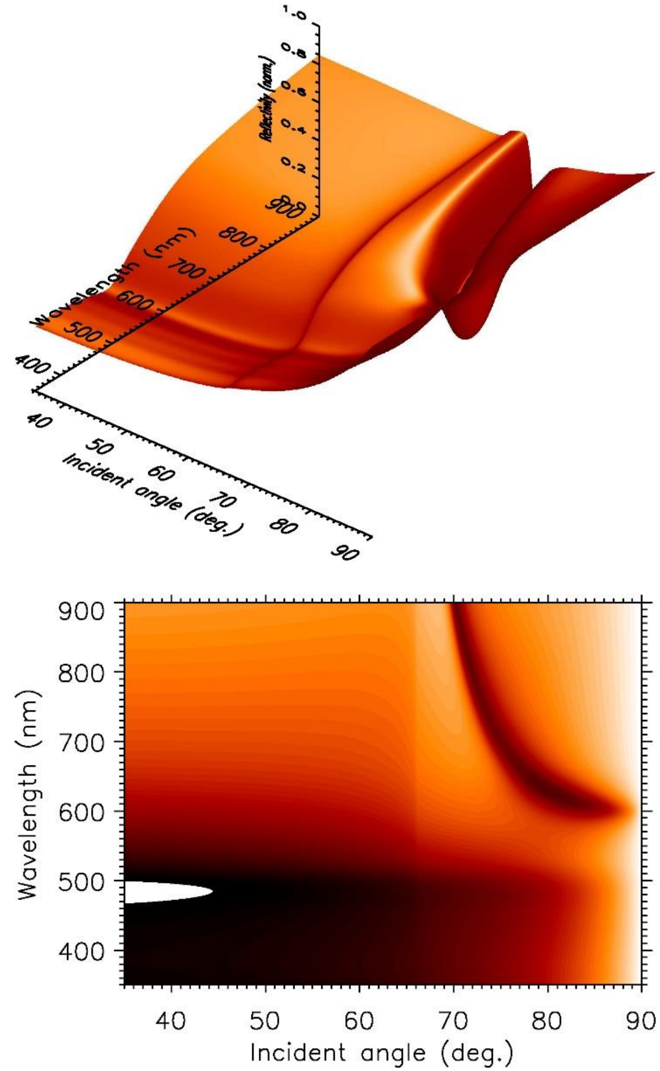


Comparison of the reflectance for LMR and SPR resonance layer structures

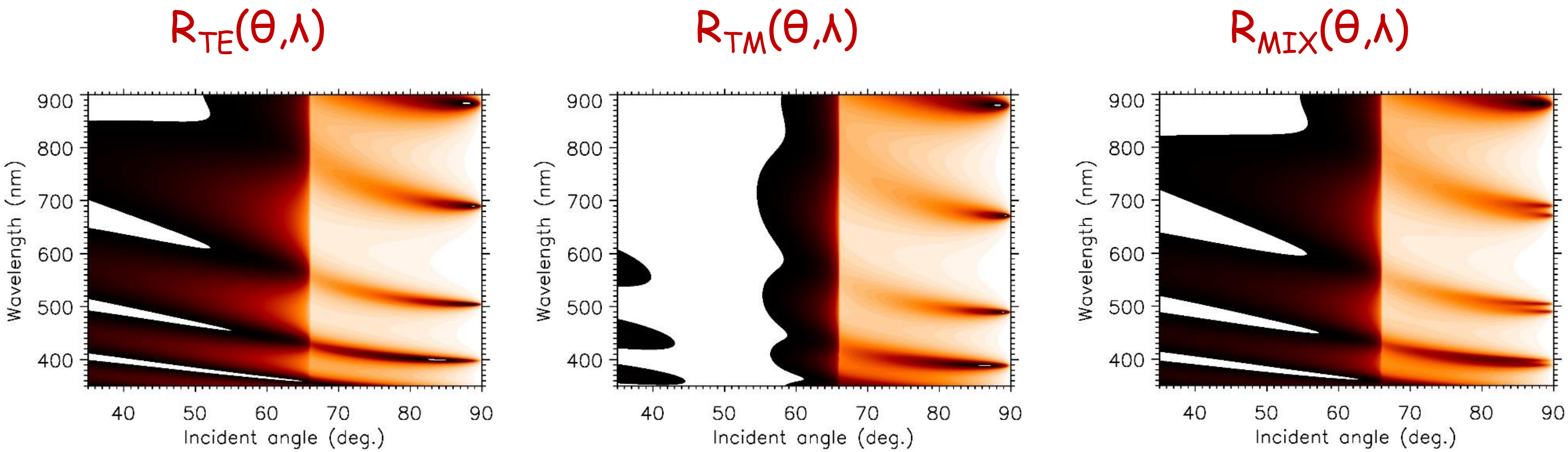
SiO_2/ITO(440 nm)/water



SiO_2/Cr(2 nm)/Au(42 nm)/water



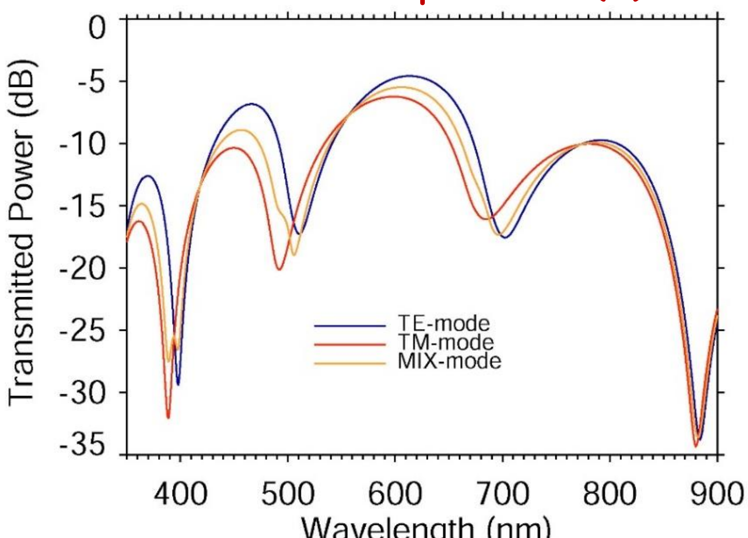
Reflectance and transmitted power for SiO2/ITO(440nm)/water



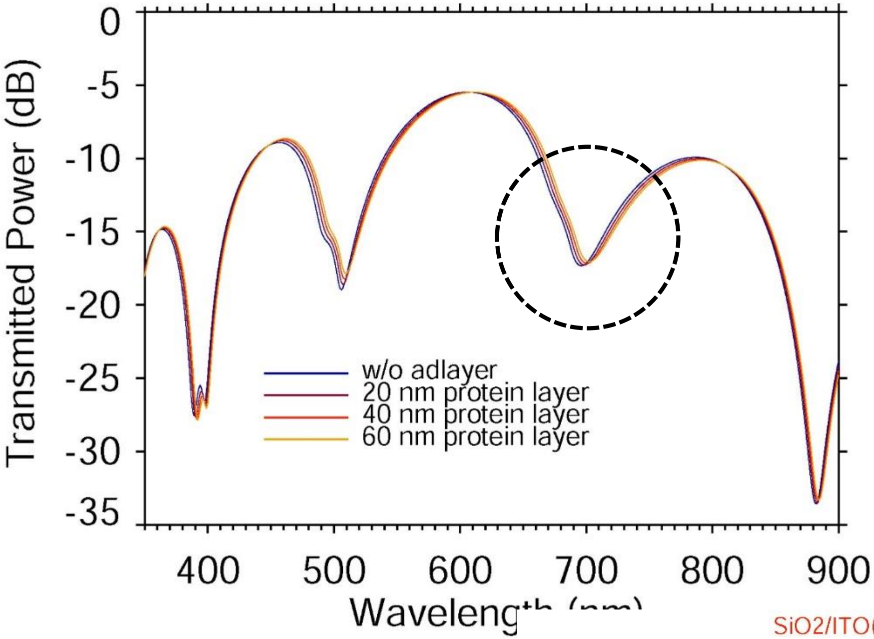
$$P = \frac{\int_{\theta_{cr}}^{\frac{\pi}{2}} R^{N(\theta)} p(\theta) d\theta}{\int_{\theta_{cr}}^{\frac{\pi}{2}} p(\theta) d\theta}$$

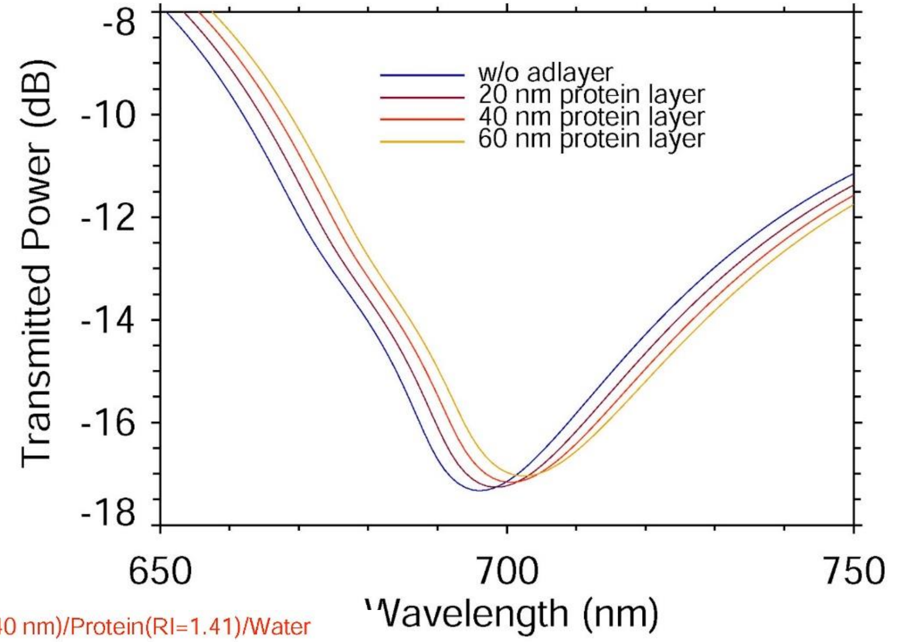
$$p(\theta) = \frac{n_1^2 \sin\theta \cos\theta}{(1 - n_1^2 \cos^2\theta)^2}$$

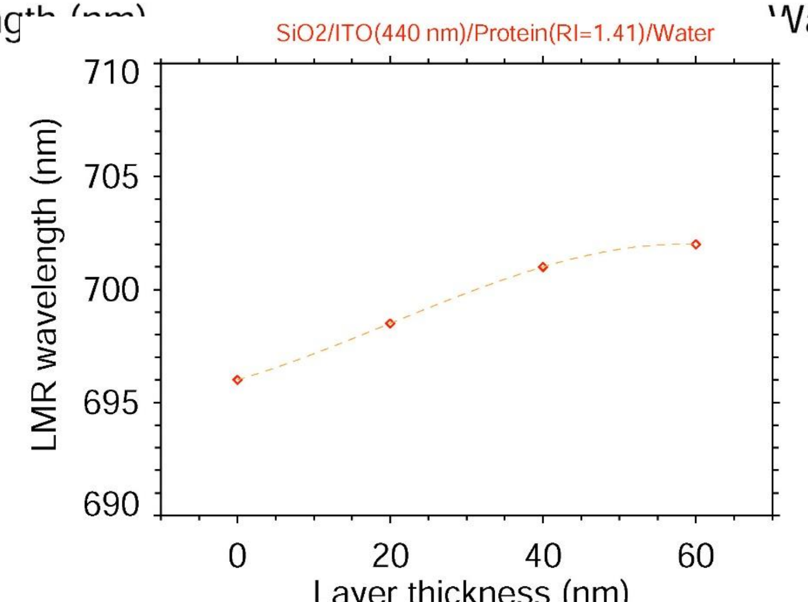
Transmitted power P(λ)



Spectral changes in the transmitted power







Conclusion

- Fiber-optic refractometer using lossy mode resonance was introduced
- Reflectance and transmitted power in a fiber were calculated and evaluated for the best performance
- The sensitivity of the sensor was estimated assuming the adsorption of protein layers with the thickness changes

Acknowledgement

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